Adaptive convergence results for numerical efficient Bayesian methods

Jan van Waaij

June 9, 2017

We observe a diffusion $X = \{X_t : t \in [0, T]\}$, which is given by the stochastic differential equation $dX_t = \theta(X_t)dt + dW_t$, where $\theta : \mathbb{R} \to \mathbb{R}$ is 1-periodic and $\int_0^1 \theta(x)^2 dx < \infty$. We are interested in estimating the unknown drift function θ. Let π be a random distribution on $\mathbb{N} \times \mathbb{R}_{>0}$ and define the random function

$$\theta^{J,S}(x) = S \sum_{j=1}^{J} \sum_{k=1}^{2^j} Z_{j,k}\psi_{j,k}(x), \quad S > 0, J \in \mathbb{N},$$

where $Z_{j,k} \overset{indep.}{\sim} N(0, 2^{-j})$ and $\psi_{j,k}$ are the Faber-Schauder basis functions. In the *hierarchical Bayes* approach we endow the parameter space with a prior as follows $\theta \mid (J, S) = \theta^{J,S}$ and $(J, S) \sim \pi$. Then the posterior distribution is given by the Bayes formula

$$\Pi(A \mid X) = \frac{\int_A p^\theta(X) d\Pi(\theta)}{\int p^\theta(X) d\Pi(\theta)},$$

where A is measurable set of the parameter space and $p^\theta(X)$ is the (Girsanov) likelihood of the data. We show optimal posterior contraction results, which adapt to the unknown smoothness of the drift function (van der Meulen, Schauer, and van Waaij 2016). An efficient numerical implementation for this prior is given in (van der Meulen, Schauer, and van Zanten 2014).

Another approach is the so-called *empirical Bayes* approach. Let $\Pi_{J,S}$ denote the distribution of $\theta^{J,S}$. We choose $(\hat{J}, \hat{S}) = \arg\max_{(J,S)\in\Lambda} \int p^\theta(X) d\Pi_{J,S}$, where Λ is a well chosen set. We then use the posterior $\Pi_{(\hat{J},\hat{S})}(\cdot \mid X)$ for the inference. In this case we also obtain adaptive convergence results. This is still ongoing work and has presumably better numerical properties.

This is collaborative work with: Moritz Schauer (Leiden University), Frank van der Meulen (Delft University of Technology) and Harry van Zanten (University of Amsterdam).

References

van der Meulen, F.H., M.R. Schauer, and J. van Waaij (2016). "Adaptive nonparametric drift estimation for diffusion processes using Faber-Schauder expansions". In: *ArXiv e-prints.* arXiv: 1612.05124 [math.ST].

van der Meulen, F.H., M.R. Schauer, and J.H. van Zanten (2014). "Reversible jump MCMC for nonparametric drift estimation for diffusion processes". In: *Comput. Statist. Data Anal.* 71, pp. 615–632. ISSN: 0167-9473. DOI: 10.1016/j.csda.2013.03.002.