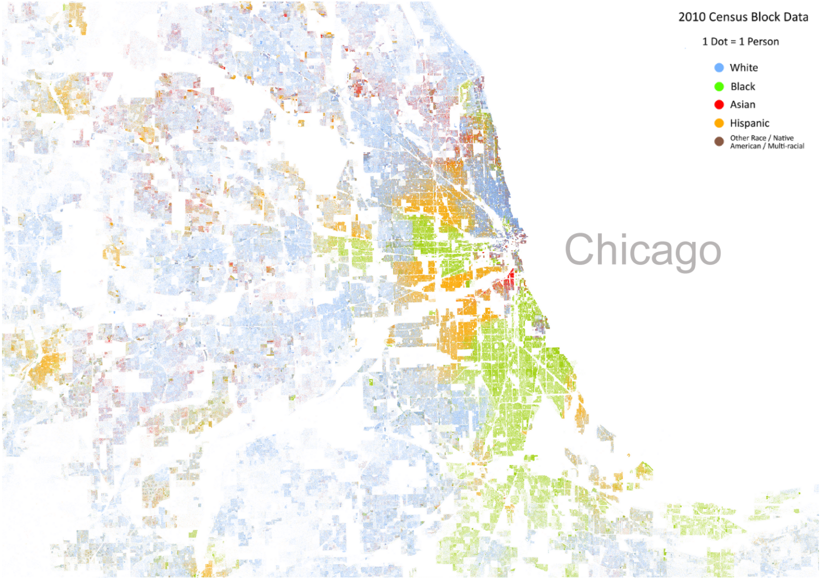


Find the interactive map at https://demographics.virginia.edu/DotMap/

Randomized Network Segregation

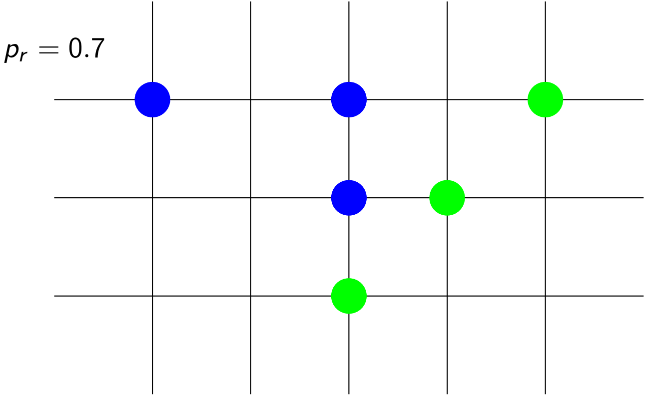
Jens Fischer

University of Potsdam

March 14, 2019

Schelling's Model

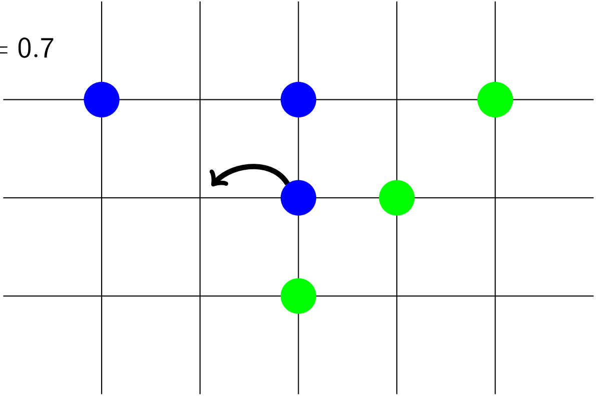
p_r minimal preferred ratio of neighbors of the same color



Schelling's Model

p_r minimal preferred ratio of neighbors of the same color

$p_r = 0.7$



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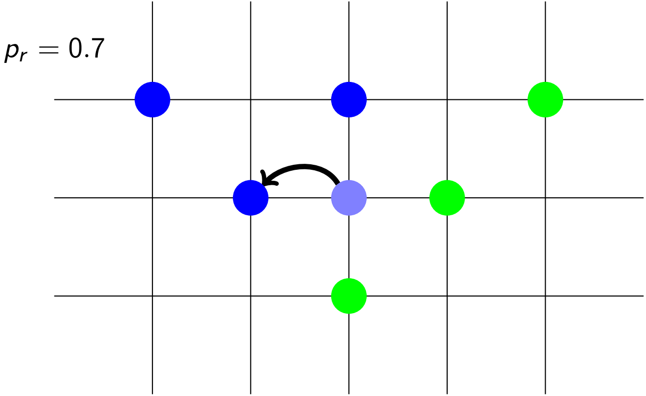

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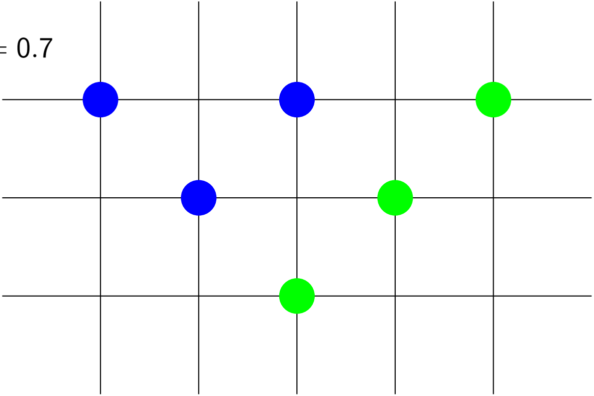
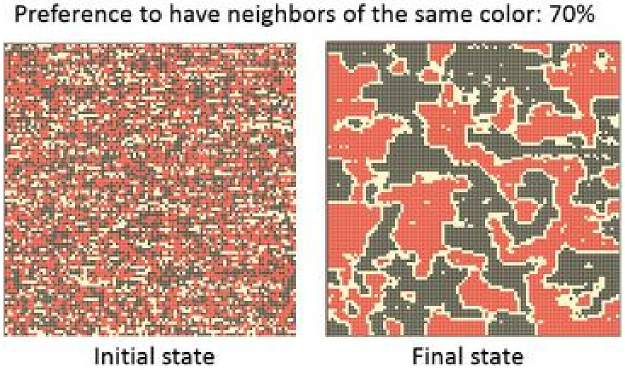
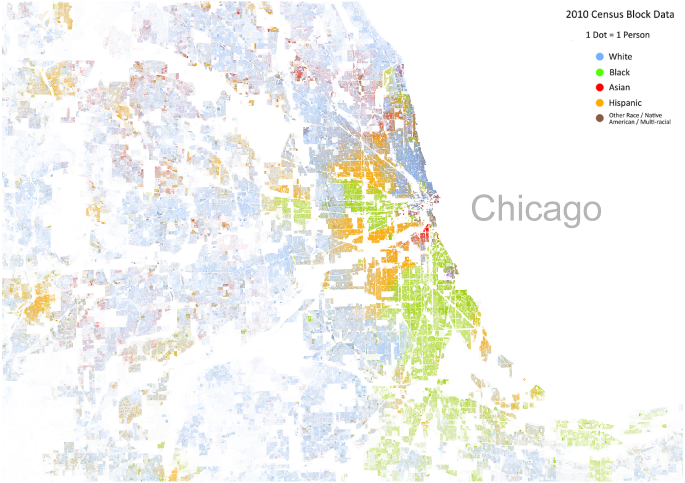


Schelling's Model



http://www.eoht.info/page/Thomas+Schelling

Segregation Phenomena



No more grids!

The Belgium Mobile Phone Network

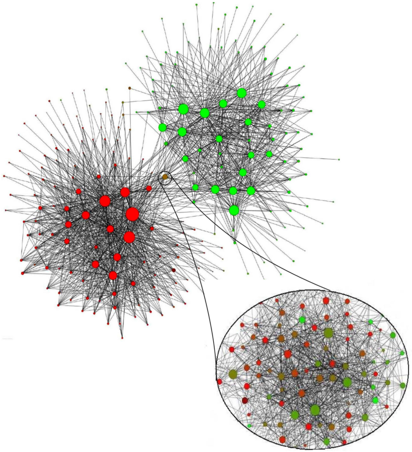
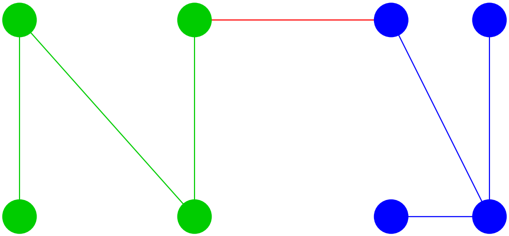


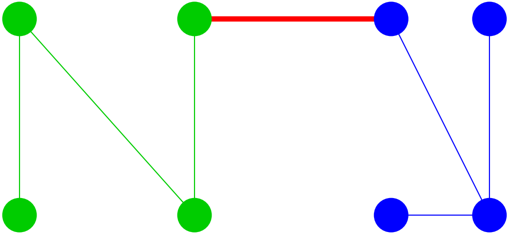
Image by Blondel, Guillaume, Lambiotte and Lefebvre (2008)

Segregation in Networks



Network segregation process inspired by Henry, Prałat and Zhang (2011)

Segregation in Networks



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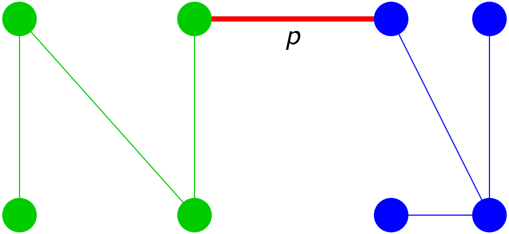

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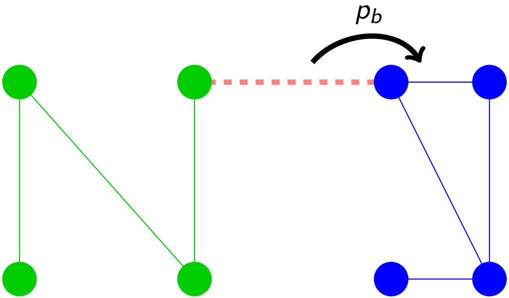

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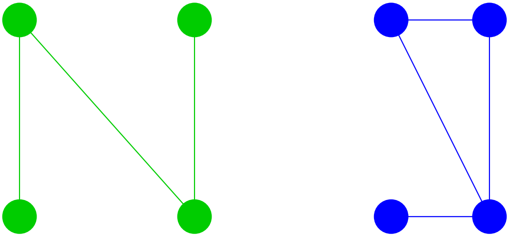


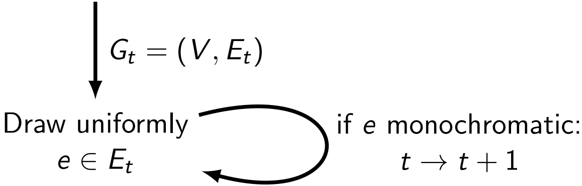
Network segregation process inspired by Henry, Prałat and Zhang (2011)

Dynamics in Algorithmic Form

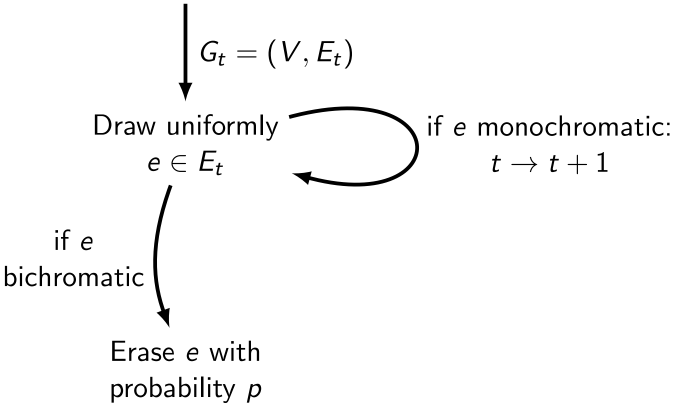
$$\Big\downarrow G_t = (V, E_t)$$

Draw uniformly
$e \in E_t$

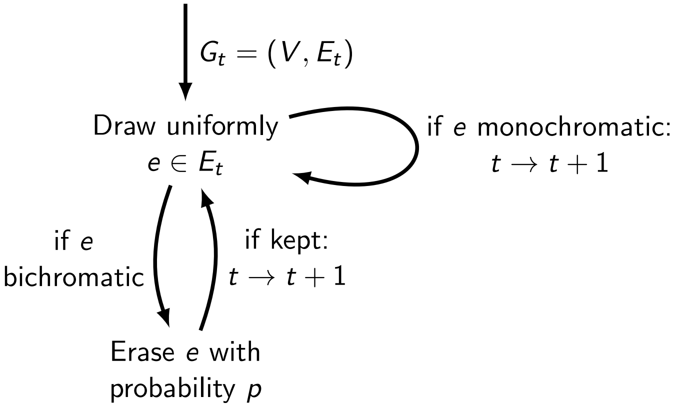
Dynamics in Algorithmic Form



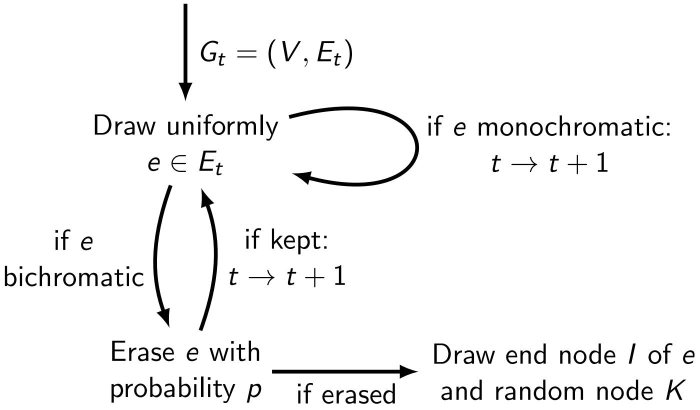
Dynamics in Algorithmic Form



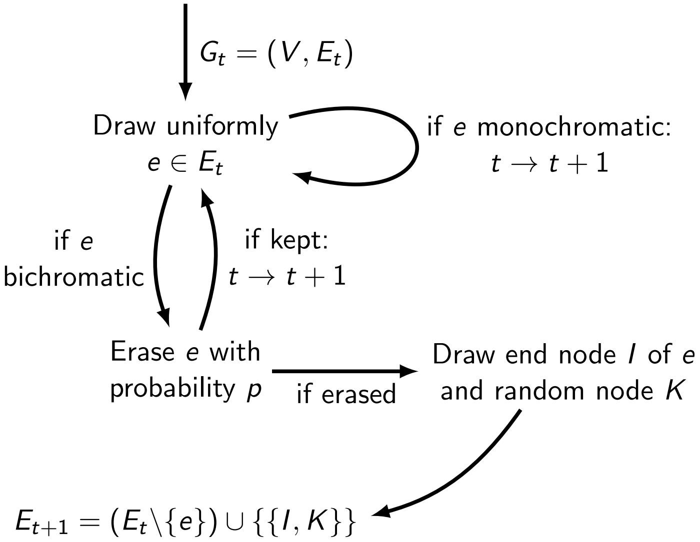
Dynamics in Algorithmic Form



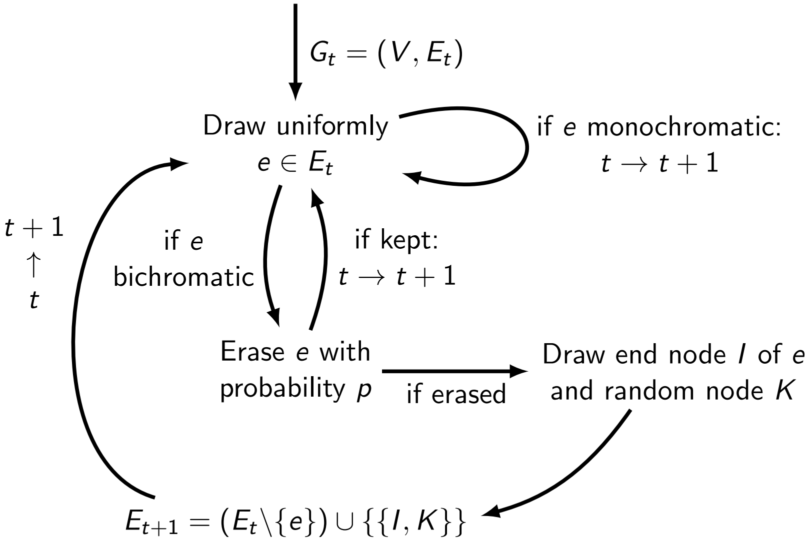
Dynamics in Algorithmic Form



Dynamics in Algorithmic Form



Dynamics in Algorithmic Form



How fast does segregation happen?

Simulation for Two Colors

$$p = 0.7$$

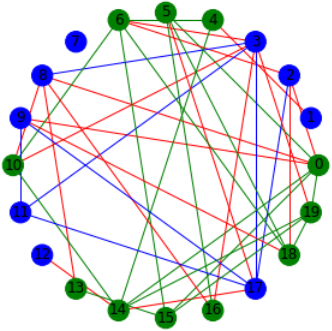


Figure: Initial state of a network

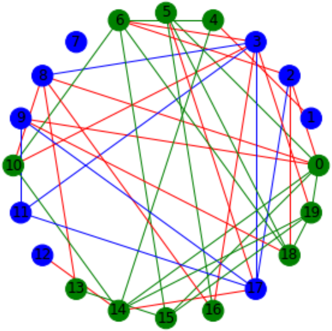
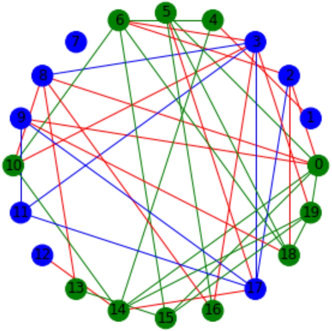


Figure: The network segregation process after 0 time steps

Simulation for Two Colors

$$p = 0.7$$



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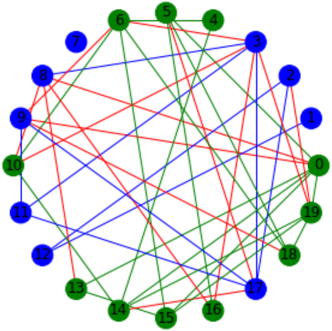
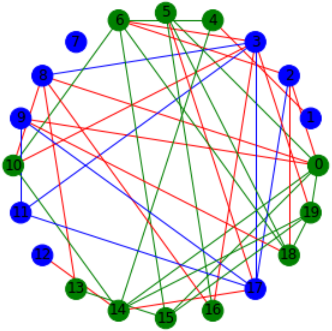


Figure: The network segregation process after 30 time steps

Simulation for Two Colors

$p = 0.7$



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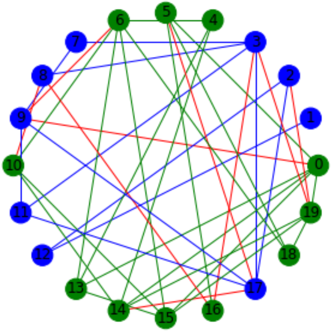

Figure: The network segregation process after 60 time steps

Simulation for Two Colors

$p = 0.7$

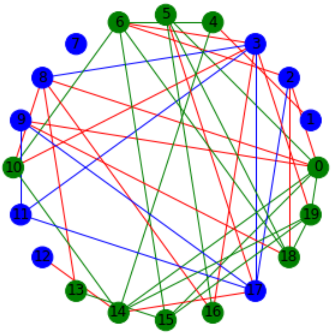

Figure: Initial state of a network

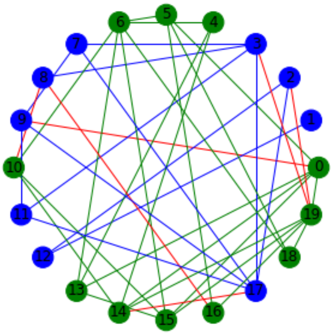

Figure: The network segregation process after 90 time steps

Simulation for Two Colors

$$p = 0.7$$

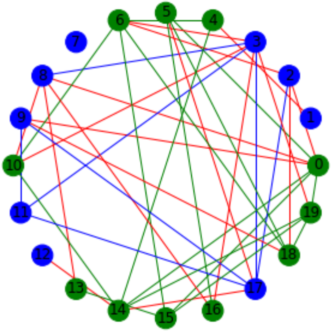

Figure: Initial state of a network

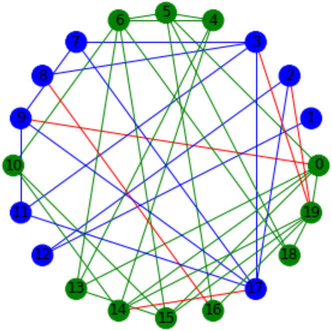

Figure: The network segregation process after 120 time steps

Simulation for Two Colors

$$p = 0.7$$

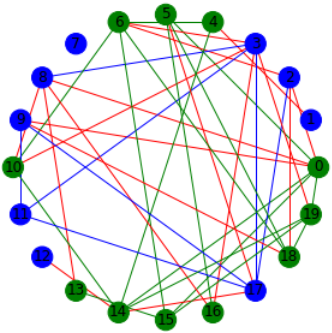

Figure: Initial state of a network

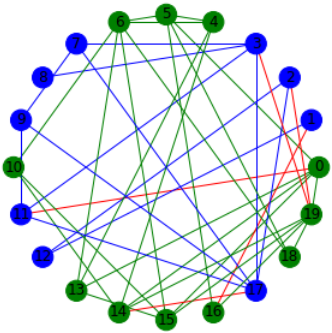

Figure: The network segregation process after 150 time steps

Simulation for Two Colors

$$p = 0.7$$

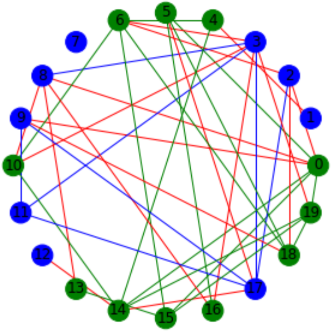

Figure: Initial state of a network

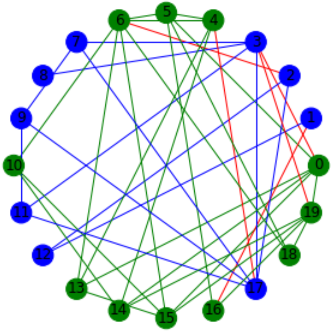

Figure: The network segregation process after 180 time steps

Simulation for Two Colors

$$p = 0.7$$

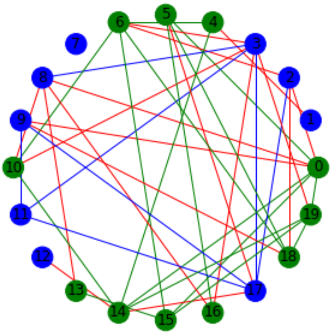

Figure: Initial state of a network

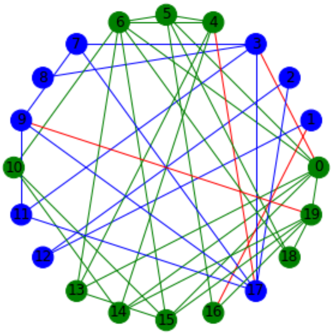

Figure: The network segregation process after 210 time steps

Simulation for Two Colors

$$p = 0.7$$

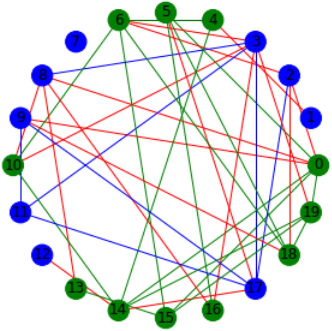

Figure: Initial state of a network

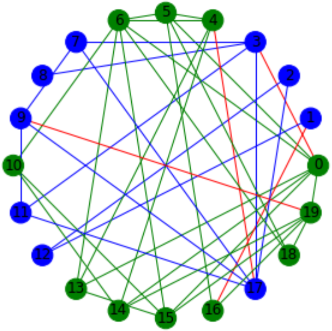

Figure: The network segregation process after 240 time steps

Simulation for Two Colors

$p = 0.7$

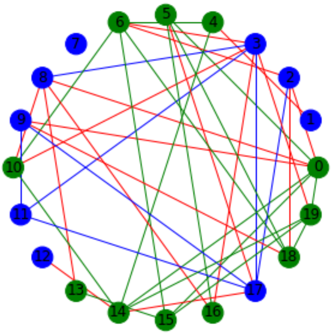

Figure: Initial state of a network

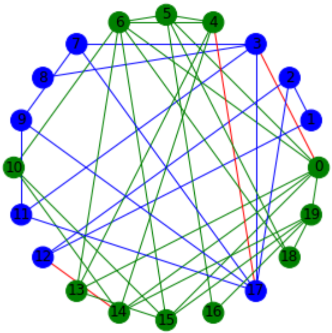

Figure: The network segregation process after 270 time steps

Simulation for Two Colors

$$p = 0.7$$

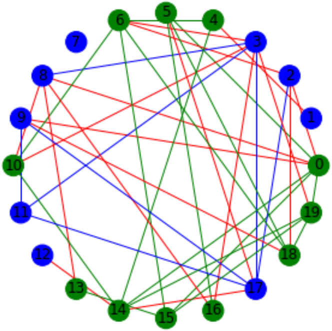

Figure: Initial state of a network

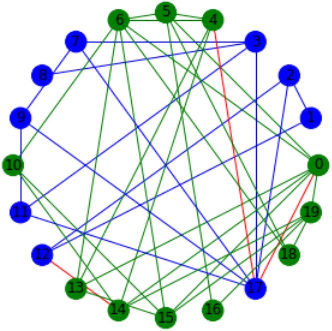

Figure: The network segregation process after 300 time steps

Simulation for Two Colors

$$p = 0.7$$

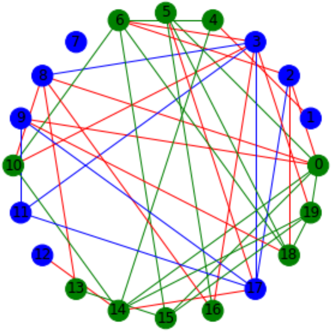

Figure: Initial state of a network

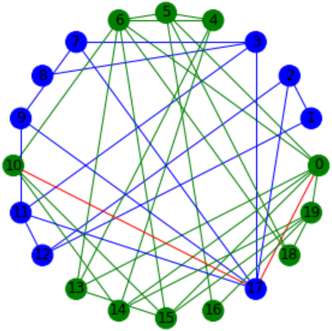

Figure: The network segregation process after 330 time steps

Simulation for Two Colors

$$p = 0.7$$

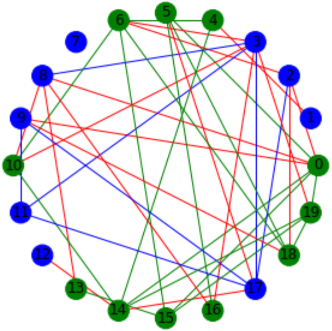

Figure: Initial state of a network

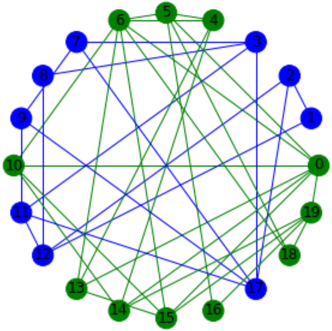

Figure: The network segregation process after 363 time steps

Multiplicative Drift in the Model

$$X_t \quad := \quad \mathrm{card}(\{e \in E_t | e \text{ bichromatic}\}), \quad t \in \mathbb{N},$$

Multiplicative Drift in the Model

$$
\begin{aligned}
X_t &:= \operatorname{card}(\{e \in E_t | e \text{ bichromatic}\}), \quad t \in \mathbb{N}, \\
T &:= \inf\{t \in \mathbb{N} | X_t = 0\}
\end{aligned}
$$

Multiplicative Drift in the Model

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\end{aligned}
$$

Properties for $s \in \{1, \ldots, m\}$

Multiplicative Drift in the Model

$$\begin{aligned} X_t &:= \mathrm{card}(\{e \in E_t | e \text{ bichromatic}\}), \quad t \in \mathbb{N}, \\ T &:= \inf\{t \in \mathbb{N} | X_t = 0\} \end{aligned}$$

Properties for $s \in \{1, \ldots, m\}$

1. $X_t - X_{t+1} \in \{0, 1\}$:

Multiplicative Drift in the Model

$$
\begin{aligned}
X_t &:= \operatorname{card}(\{e \in E_t | e \text{ bichromatic}\}), \quad t \in \mathbb{N}, \\
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Properties for $s \in \{1, \ldots, m\}$

1. $X_t - X_{t+1} \in \{0, 1\}$:
2. $\mathbb{E}[X_t - X_{t+1} | X_t = s]$

Multiplicative Drift in the Model

$$\begin{aligned} X_t &:= \mathrm{card}(\{e \in E_t | e \text{ bichromatic}\}), \quad t \in \mathbb{N}, \\ T &:= \inf\{t \in \mathbb{N} | X_t = 0\} \end{aligned}$$

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1. $X_t - X_{t+1} \in \{0, 1\}$:
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Multiplicative Drift in the Model

$$
\begin{aligned}
X_t &:= \mathrm{card}(\{e \in E_t | e \text{ bichromatic}\}), \quad t \in \mathbb{N}, \\
T &:= \inf\{t \in \mathbb{N} | X_t = 0\}
\end{aligned}
$$

Properties for $s \in \{1, \dots, m\}$

1. $X_t - X_{t+1} \in \{0, 1\}$:
2. $\mathbb{E}[X_t - X_{t+1} | X_t = s] = \mathbb{P}[X_t - X_{t+1} = 1 | X_t = s] \sim \frac{ps}{m}$

Multiplicative Drift in the Model

- $\mathbb{E}[X_t - X_{t+1} | X_t = s] \sim \frac{ps}{m}$, for $s \in \{1, \ldots, m\}$

Multiplicative Drift in the Model

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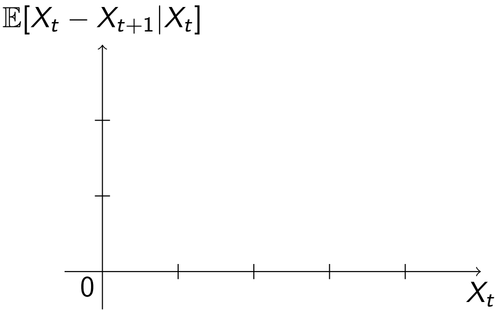

Multiplicative Drift in the Model

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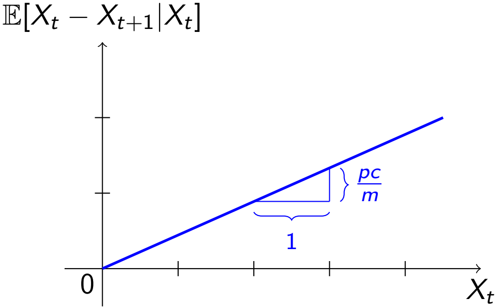

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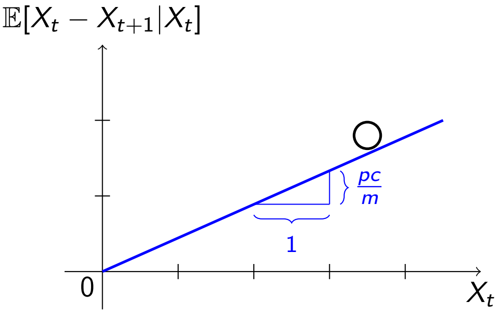

Multiplicative Drift in the Model

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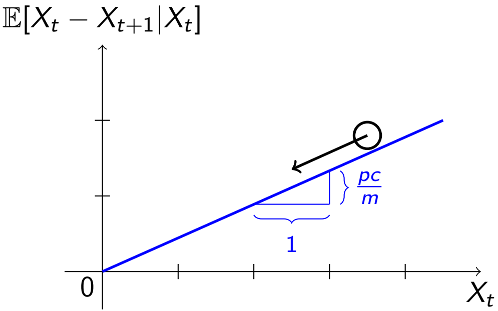

Multiplicative Drift in the Model

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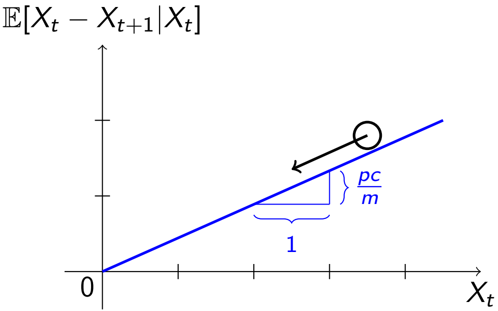
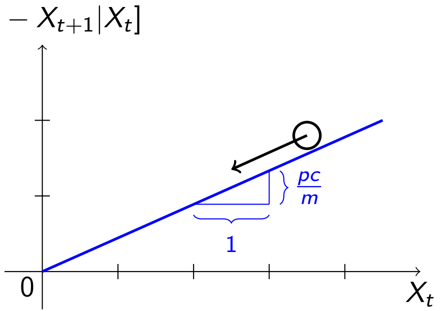


Heuristically: time $\sim \frac{\text{distance}}{\text{speed}}$

Multiplicative Drift in the Model

- $\mathbb{E}[X_t - X_{t+1} | X_t = s] \sim \frac{ps}{m}$, for $s \in \{1, \dots, m\}$



Heuristically: time $\sim \frac{\text{distance}}{\text{speed}} \Rightarrow \mathbb{E}[T|X_0] \sim \frac{m}{p} g(X_0)$

Multiplicative Drift in the Model

Theorem (Multiplicative Drift)

Let $(X_t)_{t\in\mathbb{N}}$ *be a sequence of random variables of* $\{0,1\}\cup S$ *where* $S\subset\mathbb{R}_{>1}$ *and denote by* T *the first hitting time* $T=\inf\{t\in\mathbb{N}|X_t=0\}$. *Assume that there is a* $\delta>0$ *such that for all* $s\in S\backslash\{0\}$ *and all* $t\in\mathbb{N}$

$$\mathbb{E}[X_t-X_{t+1}|X_t=s]\geq\delta s.$$

Then

$$\mathbb{E}[T]\leq\frac{1+\ln(\mathbb{E}[X_0])}{\delta}. \quad (1)$$

Further, for all $k>0$ *and* $s\in S$

$$\mathbb{P}\left[T>\frac{k+\ln(s)}{\delta}\middle|X_0=s\right]\leq\mathrm{e}^{-k}. \quad (2)$$

Result for Two Colors

Theorem

Let $\mathcal{G}^2(n, m)$ be the set of two color colored simple graphs $G = (V, E)$ with $n = |V|$ and $m = |E|$. If

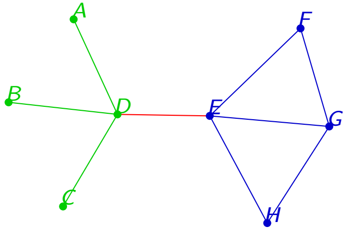
$$m \leq \frac{n^2}{64},$$

then, when starting in $\mathcal{G}^2(n, m)$, the process $(G_t)_{t \in \mathbb{N}}$ stays almost surely in $\mathcal{G}^2(n, m)$ and

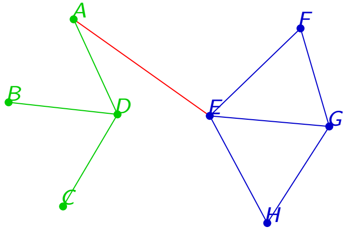
$$\mathbb{E}[T] = \mathcal{O}\left(\frac{m}{p} \log m\right)$$

such that $(G_t)_{t \in \mathbb{N}}$ segregates almost surely in finite time.

Obstacles in the Proof



Obstacles in the Proof



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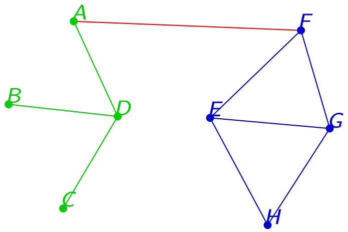

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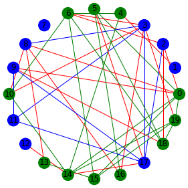

Outlook

Extend result to continuous colors (in $[-1, 1]$).

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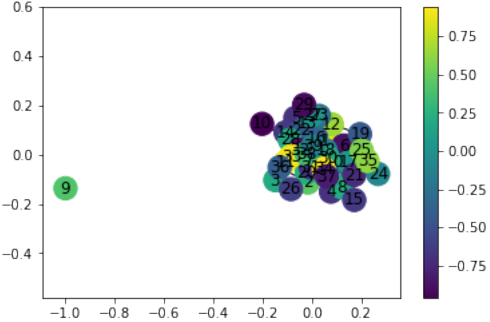
Measure similarity with distance $d : [-1, 1] \times [-1, 1] \to [0, 1]$.

Outlook

What the simulation shows:



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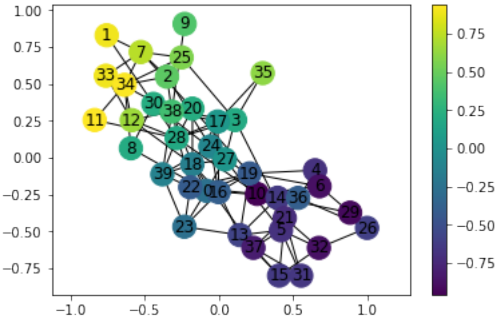

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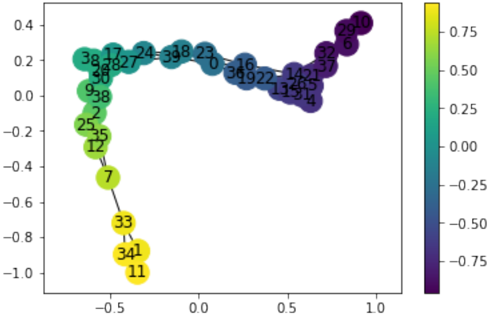

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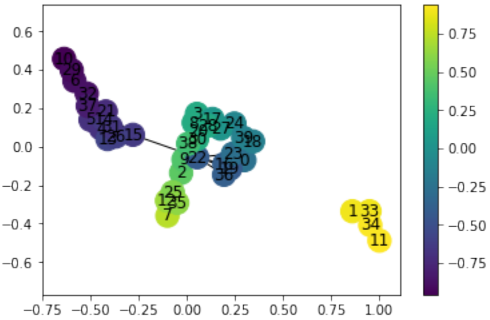

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