Andrejewski Days 2015:
100 Years of General Relativity

Schloss Gollwitz
March 2015

Bildquelle: Schloss Gollwitz

Organizers

Carla Cederbaum
(Universität Tübingen)

Jan Metzger
(Universität Potsdam)

Lecturers

Sergio Dain
(Universidad Nacional de Córdoba)

Marc Mars
(Universidad de Salamanca)

Niall Ó Murchadha
(University College Cork)

Daniel Pollack
(University of Washington)

Participants

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(Memorial University of Newfoundland)

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(University of Tennessee)

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Michael Pinkard
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Katharina Radermacher
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Marcos Ramirez
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Marcelo Rubio
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Anna Sakovich
(Albert-Einstein-Institut Golm)

Alberto Soria Marina
(Universidad de Salamanca)

Rosemberg Toala Enriquez
(University of Warwick)

Main lectures

Sergio Dain
(Universidad Nacional de Córdoba, Argentina)
Different proofs of the positive mass theorem

Marc Mars
(Universidad de Salamanca, Spain)
Rigidity results for stationary spacetimes

Niall Ó Murchadha
(University College Cork, Ireland)
Hamiltonian systems, the initial value system, and conserved quantities in General Relativity

Daniel Pollack
(University of Washington, Seattle, USA)
Initial Data for the Cauchy Problem in General Relativity

Workshop talks

Brian Allen

Inverse Mean Curvature Flow And The Proof Of The Riemannian Penrose Inequality

In this talk I will discuss Inverse Mean Curvature Flow and how it was used by Huisken and Ilmanen to prove the Riemannian Penrose Inequality. We will discuss the big ideas and calculations that go into the proof as well as make comments about recent related results.

Bernardo Araneda

Hidden symmetries and Maxwell fields on type D vacuum spacetimes

Using Killing spinors and spin reduction, we can obtain scalar equations for higher spin fields on a curved spacetime. We apply this method to Maxwell fields on Petrov type D spacetimes, with focus on the Kerr solution, and then we use adjoint operators to construct new solutions of Maxwell equations from solutions of this scalar equation. In this way, we obtain symmetry operators for both equations. We connect the results with symmetries already known, such as the Carter operator associated with a Killing tensor in Kerr spacetime.

Jose Luis Blazquez Salcedo

Rotating black holes in Einstein-Maxwell-Chern-Simons theory

We study 5-dimensional black holes in Einstein-Maxwell-Chern-Simons theory with negative cosmological constant, and free Chern-Simons coupling parameter. We consider topologically spherical black holes, with both angular momenta of equal magnitude. In particular, we study extremal black holes, which can be used to determine the boundary of the domain of existence. We compare the results of asymptotically flat solutions with the asymptotically Anti-de Sitter case. Several branches of black holes are found depending on the coupling parameters. The near horizon formalism is used to obtain some analytical results.

Week 2 (30.3.2015-3.4.2015)

Time	Monday	Tuesday	Wednesday	Thursday	Friday
8:00			*Breakfast*		
9:00 -10:30	*Lecture* Marc Mars	*Lecture* Niall Ó Murchadha	*Lecture* Marc Mars	*Lecture* Niall Ó Murchadha	*Lecture* Niall Ó Murchadha
10:45 -12:15	*Lecture* Niall Ó Murchadha	*Lecture* Marc Mars	*Excercise session* Niall Ó Murchadha	*Lecture* Marc Mars	*Excercise session* Niall Ó Murchadha
12:30			*Lunch*		
14:00-14:30			*Julien Cortier* Mass-like invariants for asymptotically hyperbolic manifolds		
14:30-15:00		*Exercise session* Marc Mars		*Exercise session* Marc Mars	
15:00-16:00	*Jonas Hirsch* Example of holomorphic functions vanishing to infinite order at the boundary		*Katharina Radermacher* The Strong Cosmic Censorship conjecture in orthogonal Bianchi B perfect fluids and vacuum		*Steve McCormick* The first law of black hole mechanics as a condition of stationarity
16:00-16:30			*Coffee Break*		
16:30-17:30	*Christopher Nerz* Constructing 'geometric coordinates' with predefined asymptotic behavior using foliations of constant mean curvature	*Ernesto Nungesser* Future of homogeneous spacetimes without cosmological constant		*Marcelo Rubio* Symplectic formalism and the covariant phase space on Scalar Electrodynamics	*Alberto Soria Marina* The Penrose inequality in Minkowski
18:00			*Dinner*		

Week 1 (23.3.2015 - 27.3.2015)

Time	Monday	Tuesday	Wednesday	Thursday	Friday
8:00			*Breakfast*		
9:00 -10:30	*Lecture* Sergio Dain	*Lecture* Sergio Dain	*Lecture* Sergio Dain	*Lecture* Sergio Dain	*Maria Eugenia Gabach Clement* On the shape of black holes
10:45-11:15			*Excercise session* Dan Pollack	*Ye Sle Cha* The Mass-Angular Momentum Inequality for Axially Symmetric Initial Data	
11:15-11:45	*Bernardo Araneda* Hidden symmetries and Maxwell fields on type D vacuum spacetimes	*Jose Luis Blazquez Salcedo* Rotating black holes in Einstein-Maxwell-Chern-Simons theory			*Oliver Lindblad Petersen* The mode solution of the wave equation in Kasner spacetimes and redshift
11:45-12:15			*Lunch*	*Lunch*	
12:30	*Lunch*	*Lunch*			*Lunch*
14:00-14:30			*Aghil Alaee Khangha* Mass functional and mass-angular momenta inequality for $U(1)^2$-invariant black holes		
14:30-15:00		*Exercise session* Sergio Dain		*Exercise session* Sergio Dain	*Exercise session* Dan Pollack
15:00-16:00	*Brian Allen* Inverse Mean Curvature Flow And The Proof Of The Riemannian Penrose Inequality		*Xián Otero Camanho* Causality Constraints on Corrections to the Graviton Three-Point Coupling		
16:00-16:30			*Coffee Break*		
16:30 -18:00	*Lecture* Dan Pollack	*Lecture* Dan Pollack		*Lecture* Dan Pollack	*Lecture* Dan Pollack
18:00			*Dinner*		

Xián Otero Camanho

Causality Constraints on Corrections to the Graviton Three-Point Coupling

We consider higher derivative corrections to the graviton three-point coupling within a weakly coupled theory of gravity. We argue that these are constrained by causality by means of a thought experiment involving a high energy scattering process. This violation cannot be fixed by adding conventional particles with spins lower or equal to two. But, it may be fixed by adding an infinite tower of extra massive particles with higher spins.

Ye Sle Cha

The Mass-Angular Momentum Inequality for Axially Symmetric Initial Data

The mass-angular momentum inequality has been proved for a large class of the axially symmetric, maximal initial data of the Einstein equation. In this talk, we will introduce how to reduce the general formulation of the mass-angular momentum inequality for non-maximal initial data, to the known maximal case, whenever a system of elliptic equations admits a solution. It is also shown that we can extend this reduction argument to the mass-angular momentum-charge inequality. The talk will be based on joint work with Marcus Khuri.

Julien Cortier

Mass-like invariants for asymptotically hyperbolic manifolds

Analogous to the asymptotically euclidean spaces, a mass has been introduced by Wang and Chrusciel-Herzlich for manifolds whose model geometry at infinity is the hyperbolic space. It also enjoys a geometric invariance property "at infinity". I will present in this talk a method to classify all such invariants, allowing various decay rates for the metric. It relies on the study of the group of asymptotic isometries. I will then discuss some geometric interpretation of them. This is based on a joint work with Mattias Dahl and Romain Gicquand.

Maria Eugenia Gabach Clement
On the shape of black holes

We discuss the description of the shape of black holes. We begin by reviewing very briefly some general aspects related to the concept of shape of ordinary objects and its extension to black holes. Then the shape of black holes in the initial and final states of black hole evolution. Finally we present some recent results in the dynamical regime. In particular we show that black hole rotation manifests in the widening of the central regions of horizons, limits their global shapes and enforces their whole geometry to be close to the extreme Kerr horizon geometry at almost maximal rotation speed. The results, which are based on the stability inequality, depend only on the horizon area and angular momentum.

Jonas Hirsch
Example of holomorphic functions vanishing to infinite order at the boundary

In general branching phenomena are of interest in geometric measure theory and geometry, and are strongly related to vanishing phenomena in the context of PDEs. An example is the analytic continuation property i.e. two holomorphic functions that agree up to infinite order at an interior point have to be identical. A more robust quantity than analyticity that captures such a property turned out to be Almgren's frequency function. For example it had been applied successful to show unique continuation for more general elliptic PDEs, (e.g. N. Garofalo, F-H. Lin), or to do a stratification procedure estimating the branch set/singular set of minimal surfaces (e.g. C. De Lellis, E. Spadaro, N. Wickramasekera et al.). Summarised, there is some literature on branching in the interior and one has unique continuation results for PDEs in the interior of their domains of definition. Little seems to be known towards the boundary. We presents examples of holomorphic functions that vanish to infinite order at points at the boundary of their domain of definition. So we give a kind of negative answer for boundary points. Moreover these example show that the monotone behaviour of Almgren's frequency function in the interior seems to be crucial. If time permits we present some implications to branching and vanishing phenomena in the context of minimal surfaces and unique continuation.

Evening talks

Saturday 28.3.2015, 19:15

Carla Cederbaum
Explaining Relativity to the Layperson?

The general public is very interested in learning about Relativity. We will discuss to what extent it is feasible to convey central ideas without relying on years of mathematical training. In particular, I will demonstrate some strategies that might help in this endeavour.

Monday 30.3.2015, 19:15

Oliver Rinne
Putting Spacetime on a Computer: Numerical Relativity

In many interesting strong-field situations, exact solutions to the Einstein equations are not available and perturbative methods do not apply. Here numerical simulations can provide helpful insights. There has been tremendous progress in recent years. I will describe the main methods used today, review some of the key achievements of numerical relativity, and conclude with some open problems.

Anna Sakovich

On the mass of asymptotically hyperbolic initial data sets

In this talk, we will focus on asymptotically hyperbolic initial data for the Einstein equations of general relativity. These objects arise naturally as hypersurfaces asymptotic to null cones in asymptotically Minkowskian spacetimes. In this case an asymptotic invariant called mass can be defined, its properties being similar to those of ADM mass of asymptotically Euclidean initial data. We will discuss perurbations which improve certain properties of asymptotically hyperbolic initial data while changing the mass arbitrarily little and will outline some recent progress towards the proof of positive mass conjecture in the asymptotically hyperbolic setting.

Marcos Ramirez

Self-gravitating splitting thin shells

We present a number of solutions of Einstein equations, in the sense of distributions, involving thin shells, and analyse their stability against separation of their constituents. This kind of solutions are important for a number of applications, like the analysis of the dynamics of globular clusters, cosmic bubbles in the early universe or brane-worlds. First, we deal with spherically symmetric shells made of Vlasov matter, and consider two different stability analysis against fragmentation: individual particle evaporation and separation of the particle ensemble into two sets. It is shown that dynamic shells may be composed by particles orbiting at different angular velocities, but in order to evolve stably as a single shell the angular momentum distribution cannot be arbitrary. In terms of the stability against separation of the particle ensemble, there are solutions that are initially stable, but turn unstable later in the evolution. In those cases a splitting solution can be constructed, where the original shell smoothly splits into a number of emergent shells. For a given initial data set, both the original shell without splitting and the splitting solution solve the Einstein equations coupled to matter, which illustrates a lack of uniqueness for the Cauchy problem. It is suggested that the unstable non-splitting solutions are not physical as they may not be thin-shell-limits of families of thick shell solutions. Finally, we extend the later stability analysis to shells composed of arbitrary non-interacting matter fields in isotropic spacetimes, with or without a cosmological constant. In particular, a SMS brane-world setting is considered, and it is shown that the same kind of instability typically appears for these models.

References: Gleiser R J, Ramirez M A, Class. Quant. Grav. 26, 045006 (2009). Gleiser R J, Ramirez M A, Class. Quant. Grav. 27, 065008 (2010). Ramirez M A, arXiv:1207.6810[gr-qc], accepted in Class. Quant. Grav. (2015).

Aghil Alaee Khangha

Mass functional and mass-angular momenta inequality for $U(1)^2$-invariant black holes

An interesting open problem is to investigate extensions of mass-angular momenta inequalities to higher dimensions. Consider a broad class of asymptotically flat, maximal slices satisfying the constraint vacuum Einstein equations in 4+1 dimensions, admitting two commuting rotational symmetries. We construct a ‘mass’ functional for t-ϕ symmetric data which agrees with the ADM mass and we show that this functional has the same critical points as Carter’s positive definite action for the stationary vacuum Einstein equations with the above spatial isometry group. The construction is a natural extension of S. Dain’s mass functional for three-dimensional initial data sets. Finally, we use this mass functional and prove a local version of a mass and angular momenta inequality for $U(1)^2$-invariant black holes.

Oliver Lindblad Petersen

The mode solution of the wave equation in Kasner spacetimes and redshift

In this talk, we consider the mode solution to the scalar wave equation $\Box\phi = 0$ in Kasner spacetimes. We present the explicit mode solution in axisymmetric Kasner spacetimes, of which flat Kasner spacetimes are special cases. For general Kasner spacetimes, we present the small and large time asymptotics of the modes. We note that in non-flat Kasner spacetimes, the modes grow logarithmically for small times, i.e. close to Big Bang. For large times, the modes oscillate with a polynomially decreasing amplitude. This gives a notion of large time frequency of the modes, which we use to model the wavelength of light rays in Kasner spacetimes. We show that the redshift one obtains by modelling light as a mode solution of the wave eqaution actually coincides with the usual cosmological redshift.

Christopher Nerz

Constructing 'geometric coordinates' with predefined asymptotic behavior using foliations of constant mean curvature

In mathematical general relativity, one often assumes that the space-time is foliated by space-like hypersurfaces such that each of these surfaces satisfies certain asymptotic assumptions. The latter are often defined using coordinates. For example, isolated gravitational systems are modeled by space-times which are foliated by 'asymptotically flat manifolds', i. e. it is assumed that each leaf M possesses a coordinate system x mapping M (outside some compact set) to the Euclidean space (outside some ball). Using this type of assumption, a physical property is modeled using a non-geometric one which seems to be counterintuitive. We can resolve this by constructing 'geometric coordinate systems' using 'geometric spheres', i.e. characterizing the above asymptotic assumptions in a coordinate-free way. In this talk, we explain this by considering asymptotically flat and asymptotically hyperbolic manifolds and spheres of constant mean curvature.

Ernesto Nungesser

Future of homogeneous spacetimes without cosmological constant

I will present different results concerning future stability of solutions to the Einstein-Vlasov system with Bianchi symmetry. In particular I will present a new result which represents an analogue to the asymptotic self-similar breaking in the Einstein-Euler case.

Katharina Radermacher

The Strong Cosmic Censorship conjecture in orthogonal Bianchi B perfect fluids and vacuum

Einstein's equation in General Relativity can be formulated as an initial value problem, where the initial data consists of the metric and second fundamental form on a three-dimensional Cauchy hypersurface. Choquet-Bruhat proved that this initial value problem has a maximal globally hyperbolic development which is unique up to isometry. That this development is inextendible, at least for generic initial data, is the statement of the Strong Cosmic Censorship conjecture.

In this talk, I will consider the case where the Cauchy hypersurface is a three-dimensional non-unimodular Lie group (i.e. a Bianchi class B model) and the stress energy tensor that of a perfect fluid or vacuum. I will sketch a proof of this conjecture and state several additional properties regarding asymptotic behaviour towards the initial singularity.

Marcelo Rubio

Symplectic formalism and the covariant phase space on Scalar Electrodynamics

joint with Oscar Reula (Universidad Nacional de Córdoba)

In this talk I will make a review of the covariant phase space formalism on field theory (Refs. [1, 2]) and an application on scalar classical electrodynamics. This formalism consists on taking a infinite dimensional manifold in which each point is a solution of field equations (that is, each point represents the entire history of the system) and it is equipped with a closed two-form Ω, the pre-symplectic structure. Degenerate directions of Ω are the infinitesimal gauge transformations of the theory and can be shown to be integrable. A notion of symmetry can be constructed from this formalism, and thus obtain conserved quantities associated with them. I will discuss classical scalar electrodynamics from this point of view, and thus recover symmetries and their respective conserved charges.

References

[1] A. Ashtekar; L. Bombelli and O. Reula. The Covariant Phase Space of Asymptotically Flat Gravitational Fields. In Mechanics, Analysis, and Geometry: 200 Years After Lagrange, edited by M Francaviglia, 118. Elsevier Science Ltd, 1991.

[2] C. Crnkovic and E. Witten. Covariant description of canonical formalism in geometrical theories. In Three Hundred Years of Gravitation, edited by S. W. Hawking and W. Israel, pp. 676684, 1987.

Steve McCormick

The first law of black hole mechanics as a condition of stationarity

The first law of black hole mechanics states that for infinitesimal perturbations to a stationary black hole, a differential relationship between various physical quantities must be satisfied this is analogous to the first law of thermodynamics for bodies in equilibrium. In 1992, Sudarsky and Wald presented an argument suggesting a converse to this statement [Phys. Rev. D 46, 1453]; it was argued that if infinitesimal perturbations to a given black hole satisfy the differential relationship given by the first law, then it should indeed be stationary.

In this talk, we discuss recent work that establishes a rigorous proof of this result [Phys. Rev. D 90, 104034]. We describe the phase space for the Einstein-Yang-Mills equations using weighted Sobolev spaces, discuss the relevant physical quantities in the first law, and outline the Lagrange multiplier argument used to establish the key result.

Alberto Soria Marina

The Penrose inequality in Minkowski

The Penrose inequality in Minkowski is a geometric inequality relat- ing the total outer null expansion and the area of closed, connected and spacelike codimension-two surfaces $\mathcal{S}$ in the Minkowski spacetime, subject to an additional convexity assumption. The validity of this inequality still remains open. In this work we analyze the problem and prove the inequality in special cases.