Max-Planck-Institut für
Gravitationsphysik
(**Albert-Einstein**-Institut)

CAUSALITY CONSTRAINTS

on corrections to the graviton 3-point coupling

March 25, 2015
ANDREJEWSKI days @ Schloss Gollwitz

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Potsdam-**Golm**

[**1407.5597** & to appear]
based on joint work with J. Edelstein, J. Maldacena & A. Zhiboedov

MOTIVATION

- Understand **classical** field theories (weakly coupled)
- **Consistency** conditions on classical lagrangians
  - unitarity
  - Lorentz invariance

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What is... ?

1. most general »pure gravity« theory?
   (only **massless gravitons**)
2. most general »classical« gravity theory?
   (possibly including massive **higher spins**)

OUTLINE

1. Perturbative (Q)FT & Feynman diagrams
2. Causality
3. Journey through the shock
4. Higher-spin fix
5. Conclusions

PERTURBATIVE (Q)FT & FEYNMAN DIAGRAMS

GEOMETRY VS. QFT

We can look at **gravitational interactions** in two complementary ways

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- **QFT**

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Weinberg '64

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    $h_{\mu\nu}$ / $p^\mu h_{\mu\nu} = h^\mu_{\ \mu} = 0$ (5? d.o.f.)

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$$h_{\mu\nu} \rightarrow h_{\mu\nu} + \alpha_{\mu} p_{\nu} + \alpha_{\nu} p_{\mu} \qquad ; \quad \alpha^{\mu} p_{\mu} = 0$$

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  - **Equivalence** *theorem* $\qquad \sum_i \kappa_i p_i^\mu = 0 \quad \Rightarrow \quad \kappa_i = \kappa$

- Many quantities can be computed in both frameworks.

FEYNMAN DIAGRAMS IN FIELD THEORY

Klein-Gordon equation

$(\nabla^2 + m^2)\phi = 0 \quad ; \quad \phi = e^{-ipx}$ with $p^2 = m^2$ *(on-shell)*

$\rightarrow$ it is **linear:** $\phi_0 = \int dp\, \delta(p^2 - m^2) \left[a(p)\, e^{-ipx} + a^*(p)\, e^{ipx}\right]$

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Nonlinear equation

$(\nabla^2 + m^2)\phi = g\phi^3$; $\phi = \sum_n g^n \phi_n$

→ Green's function: $(\nabla^2 + m^2)\, G(x, y) = \delta(x - y)$

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(propagator) $G(x, y) = \int \frac{dk}{(2\pi)^d} \frac{e^{-ik(x-y)}}{-k^2 + m^2}$ *(off-shell)*

$(\nabla^2 + m^2)\phi_1 = \phi_0^3$; $\phi_1(x) = \int dy\, G(x, y)\phi_0^3(y)$

$(\nabla^2 + m^2)\phi_2 = 3\phi_0^2\phi_1$; $\phi_2(x) = 3 \int dy\, G(x, y)\phi_0^2(y)\phi_1(y)$

⋮

diagramatic expansion!

CAUSALITY

CAUSALITY & QFT

Not all local, Lorentz invariant lagrangians are consistent.

Adams, Arkani-Hamed, Dubovsky, Nicolis & Rattazzi '06

e.g. massless scalar field

$$\mathcal{L} = -\partial_\mu \phi \, \partial^\mu \phi + \frac{c}{\Lambda^4} \left(\partial_\mu \phi \, \partial^\mu \phi\right)^2 + \dots$$

Effective metric: $\phi = \phi_0 + \psi, \quad \partial_\mu \phi_0 = C_\mu$

$$\underbrace{\left(\eta^{\mu\nu} - 4\frac{c}{\Lambda^4} C^\mu C^\nu + \dots\right)}_{G^{\mu\nu}(\phi_0)} \partial_\mu \partial_\nu \psi + \dots = 0$$

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In order to avoid **causality violations**: $\boxed{c \geq 0}$ (and **CTCs**)

Assuming **analiticity** & **unitarity**:

$$\frac{c}{\Lambda^4} = \frac{2}{\pi} \int ds \, \frac{\sigma(s)}{s^2} \geq 0$$

CAUSALITY & GRAVITY

QFT: global Lorentz symmetry (Lorentz inv. notion of causality)
Gravity: less obvious

- just **local** Lorentz invariance
- locally $c_g > 1$ not necessarily leads to **CTCs**

Lorentz invariance still **asymptotic symmetry** Gao & Wald '00

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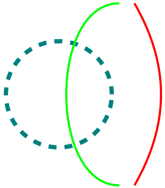
$\Rightarrow$ **asymptotic causality**

- Null energy condition
- Einsteins' equations



We can prove the positivity of mass in this way Penrose, Sorkin, Woolgar
and can be generalised to asymptotically AdS spacetimes
(**holographic causality**) Page, Surya, Woolgar
Brigante ***et al.***

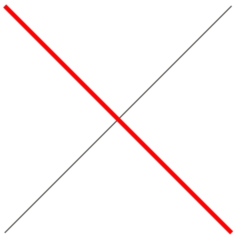
JOURNEY THROUGH THE SHOCK

SHAPIRO TIME DELAY

4-th **classical test** of GR: light slowed down by the gravitational field of a massive body.

Simplified experiment: probe in the gravitational field of a highly energetic particle (**shock wave**)

$$ds^2 = -du\,dv + \delta(u)h(x)\,du^2 + dx^i dx^i \qquad ; \qquad h(|x|) = G_N \frac{|P_u|}{|x|^{d-4}}$$



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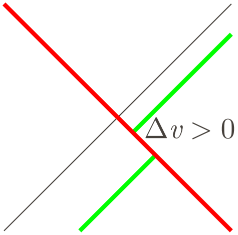


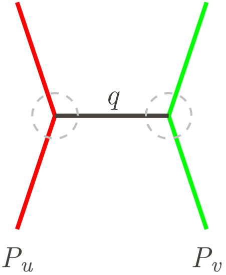
probe **delay**

$$\Delta v = h(b) > 0$$

same as for a **scalar** field or **GR** (phase shift) $\quad \delta = P_v \Delta v$

3-POINT FUNCTIONS

There is a equivalent description in terms of 3pt functions



The Mandelstam invariants:

$$s = P_u P_v\,, \qquad t = -q^2$$

Forward limit, $s \gg t$

$$\mathcal{A}_{tree}(s,t) = \frac{s^2}{t}$$

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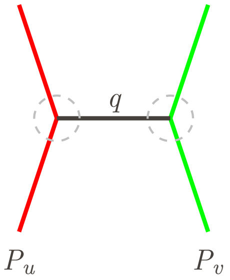

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Eikonal approximation (**impact parameter** representation)

$$\delta(s,b) \;=\; \frac{1}{s}\int d^{d-2}\boldsymbol{q}\, e^{i\boldsymbol{b}\cdot\boldsymbol{q}} \mathcal{A}_{tree}(s,-q^2) = G_N \frac{s}{b^{d-4}} \equiv -P_v \Delta v$$

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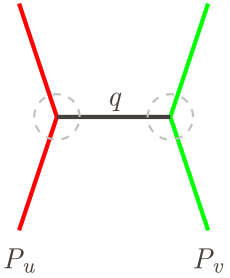

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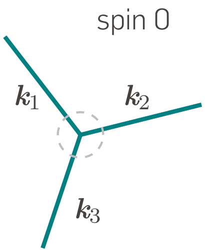
Eikonal approximation & **factorization** (**massless** pole)

$$\begin{aligned} \delta(s,b) &= \frac{1}{s} \int d^{d-2}\boldsymbol{q}\, e^{i\boldsymbol{b}\cdot\boldsymbol{q}} \mathcal{A}_{tree}(s,-q^2) \\ &= \frac{1}{s} \sum_i \mathcal{A}_3^i(\boldsymbol{q}=\partial_{\boldsymbol{b}}) \mathcal{A}_3^i(\boldsymbol{q}=\partial_{\boldsymbol{b}}) \frac{1}{b^{d-4}} \end{aligned}$$

3-POINT FUNCTIONS II



No kinematic invariants:
$(\boldsymbol{k}_1 + \boldsymbol{k}_2)^2 = \boldsymbol{k}_3^2 = 0$

Only a single **coupling** $\sqrt{G_N}$

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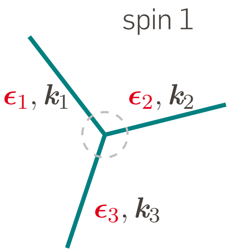

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$\boldsymbol{k}_i \cdot \boldsymbol{\epsilon}_i = 0 \quad ; \quad \boldsymbol{\epsilon}_i \sim \boldsymbol{\epsilon}_i + \boldsymbol{k}_i$

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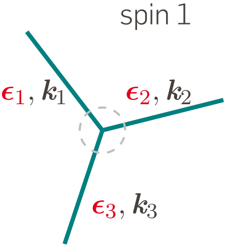

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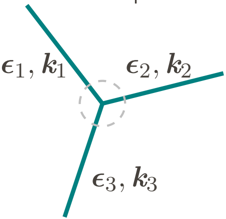
$$
\begin{aligned}
A_0 &= (\boldsymbol{\epsilon}_1 \cdot \boldsymbol{\epsilon}_2)(\boldsymbol{\epsilon}_3 \cdot \boldsymbol{k}_1) + \dots && \sim E \ (F^2) \\
A_2 &= (\boldsymbol{\epsilon}_1 \cdot \boldsymbol{k}_2)(\boldsymbol{\epsilon}_2 \cdot \boldsymbol{k}_3)(\boldsymbol{\epsilon}_3 \cdot \boldsymbol{k}_1) && \sim E^3 \ (F^3)
\end{aligned}
$$

GRAVITY 3-POINT FUNCTIONS

spin 2



$$G_0 = A_0 A_0 \qquad \frac{1}{G_N} R$$

$$G_2 = A_0 A_2 \qquad \frac{\alpha_2}{G_N} \mathcal{R}^2$$

$$G_4 = A_2 A_2 \qquad \frac{\alpha_4^2}{G_N} \mathcal{R}^3$$

$$\alpha_{2,4} \sim [L^2]$$

extra terms **relevant** at distances $r \sim \sqrt{\|\alpha_{2,4}\|}$

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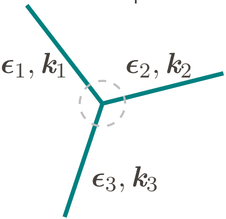

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Effective field theory: $\alpha_{2,4} \sim l_p^2$ (**strong** coupling)

Weakly coupled gravity: $\alpha_{2,4} \gg l_p^2$

Overall coupling G_N very small (**all** three very **small**)

e.g. string theory $g_s \to 0\,,\quad \alpha_{2,4} \sim \alpha'$

WEAKLY COUPLED GRAVITY THEORIES

Consider a general gravity theory

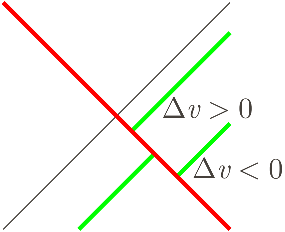
$$\frac{1}{16\pi G_N}\int d^d\boldsymbol{x}\sqrt{-g}\left(R+\alpha_2\mathcal{R}^2+\alpha_4^2\mathcal{R}^3+\ldots\right)$$

and compute the **time delay** (for a scalar source)

$$\begin{aligned}\Delta v &= \left(1+\alpha_2(\boldsymbol{\epsilon}\cdot\partial_{\boldsymbol{b}})^2+\alpha_4^2(\boldsymbol{\epsilon}\cdot\partial_{\boldsymbol{b}})^4\right)\frac{G_N||P_u||}{b^{d-4}}\\ &= \left(1\pm\frac{\alpha_2}{b^2}\pm\frac{\alpha_4^2}{b^4}\right)\frac{G_N||P_u||}{b^{d-4}}\end{aligned}$$

SHAPIRO TIME DELAY

Depending on the **polarization** we can propagate **faster than light** as seen from infinity.



Violates **asymptotic causality**

Gao & Wald '00

In **AdS**, it corresponds to a violation in the **boundary theory**.

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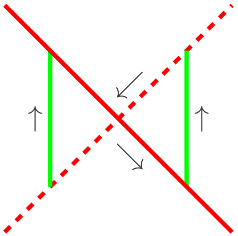

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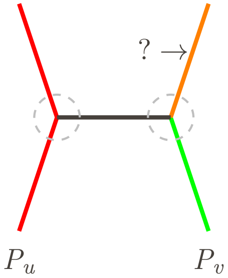
Indication for the existence of **CTCs**

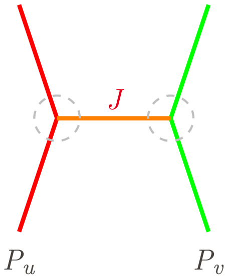
HIGHER-SPIN FIX

CORRECTING THE PROBLEM



Adding more **external particles** does not help

CORRECTING THE PROBLEM



Graviton contribution grows like

$$s = P_u P_v$$

Contribution from **spin** J particles grows like s^{J-1}

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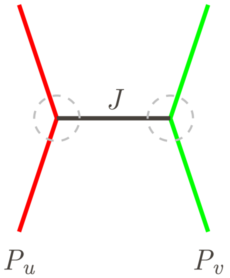

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$$J \geq 2 \quad ; \quad m_J^2 \lesssim \frac{1}{\alpha_{2,4}}$$

Massive **spin two** does not help

Massive **higher spins** have problems with analyticity
$\Rightarrow$ we need an **infinite** number!

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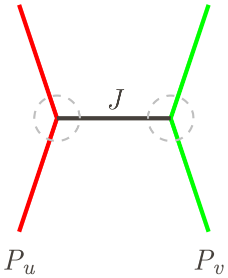

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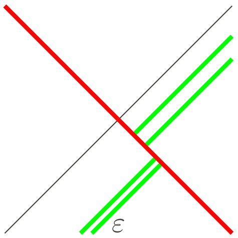
Massive **higher spins** have problems with analyticity
$\Rightarrow$ we need an **infinite** number! $\rightarrow$ **it works for strings!!**

Amati, Ciafaloni, Veneziano '88

EXTENDED GRAVITON AND 3-POINT FUNCTIONS



If the **graviton is extended** different ***pieces*** suffer different time delays:

$$\varepsilon \sim l_s$$

$$\frac{1}{2}\left[\delta(b+\varepsilon)+\delta(b-\varepsilon)\right] \approx \delta(b) + \frac{\varepsilon^2}{2}\partial_b^2\delta(b)$$

from where: $\boxed{\alpha_2 \sim l_s^2}$ $\sim$ typical length of the ***string***

CONCLUSIONS

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Danke schön!!