

# Monday

## **Periodic Jacobi matrices on trees**

Jonathan Breuer (Hebrew University)

## **Eigenvalues of graph Jacobi matrices and spectra of periodic covers**

Eyal Seelig (Hebrew University)

Classical bounds in spectral graph theory, e.g. Alon-Boppana, display a relation between eigenvalues of the adjacency matrix of finite graphs to the spectrum of its universal cover. In this talk we show several generalizations of these ideas to Jacobi matrices on graphs. It is a part of an ongoing research project aimed at periodic Jacobi matrices on trees.

## **On the spectral analysis of Schrödinger operators on star-like graphs**

Netanel Levi (Hebrew University)

The study of Schrödinger operators on graphs has drawn a considerable amount of attention in the past few decades. For such operators on the half-line, Gilbert and Pearson (1987) introduced a way to study the spectral continuity properties via asymptotic properties of solutions to the eigenvalue equation. In this talk, we present a generalization of this method to star-like graphs, which are given by attaching a finite number of half-lines to a compact graph.

## **Resolvable Dirichlet spaces**

Simon Puchert (Friedrich-Schiller-University Jena)

In 1996, Benjamini and Schramm introduced the concept of resolvable graphs and proved the existence of non-constant bounded harmonic functions of finite energy on transient resolvable graphs. These ideas can be generalized to apply to Dirichlet spaces and as a nice side effect, the proofs can be simplified considerably. The talk is based on joint work (in progress) with Daniel Lenz and Marcel Schmidt.

## **Philippe Charron (Université de Genève)**

Zeroes of linear combinations on quantum graphs

Sturm's oscillation theorem states that the  $N$ -th eigenfunction of a Sturm-Liouville operator on an interval has exactly  $N-1$  zeroes. A much lesser-known result that he published in the same year states that any linear combination of eigenfunctions from  $M$  to  $N$  has between  $M-1$  and  $N-1$  zeroes. On manifolds, Courant's theorem says that the  $N$ -th eigenfunction has at most  $N$  nodal domains, but there are example of manifolds with linear combinations with infinitely many nodal domains. In this talk, I will discuss the situation for graphs, which are in a sense "halfway" between intervals and manifolds.

# Tuesday

## **About endomorphisms of B-free, B-admissible subshifts**

Daniel Sell (Torun)

Subshifts are dynamical systems, whose elements are infinite strings of symbols. We will look at definitions and examples of two ways to define them from sets of natural numbers, leading to so-called B-free and B-admissible subshifts. We show that every continuous, shift-compatible map on these systems is monotone. As a consequence, if such a map is either surjective or pre-injective, then it is already bijective (and, in fact, a power of the shift). The talk is based on joint work with Gerhard Keller (Erlangen), Mariusz Lemańczyk (Toruń) and Christoph Richard (Erlangen).

## **Periodic approximation of substitution subshifts**

Lior Tenenbaum

In studying higher dimensional Schrödinger operators of quasicrystals, one is led to find suitable periodic approximations. This means in particular that the spectrum converges as a set to the limiting spectrum. It turns out that the convergence of the underlying dynamical systems is exactly what one needs to do so. This is the starting point of the present talk.

We treat subshifts defined through so-called substitutions. These subshifts provide models of aperiodic ordered systems. We find natural sequences of subshifts converging to the substitution subshift. Well-known examples of substitution subshifts are discussed during the talk. We will also discuss the motivation for this characterization, arising from an attempt to study higher dimensional quasi-crystals. This is based on a Joint work with Ram Band, Siegfried Beckus and Felix Pogorzelski.

## **On Characterization of Gromov hyperbolic spaces and their applications**

Robert Müller (University of Potsdam)

The notion of curvature is well-known for Riemannian manifolds. Furthermore one can consider Riemannian manifolds as metric spaces. In this talk we want to abstract the notion of negative curvature for general metric spaces and characterize this property of so called Gromov hyperbolic spaces.

## **Laplacians and spectral geometry on simplicial complexes**

Philipp Bartmann (University of Potsdam)

TBA

## **Dynamical spectrum and diffraction theory for subshifts**

Anna Rockstroh (Friedrich-Schiller-University Jena)

Mathematical diffraction theory has its origins in physics, in particular in the study of diffraction patterns of quasi crystals. However, the theory can also be applied to other, more abstract objects such as dynamical systems. The dynamical system which we will consider is a subshift. The aim is to draw a connection between diffraction theory and spectral theory of the subshift: We will recognize that the spectrum of the subshift is pure point if and only if its diffraction is a pure point measure. This work was supervised by Daniel Lenz.

## **A comparison of two theorems for quasicrystals**

Marc Vieth (University of Leipzig)

Since the discovery from quasicrystals it is always an interesting questions what are the properties which give me a Crystal instead of a quasicrystals. Lev and Olevski dedicated this question to one of their papers and got a nice result. A similar Result was also reached by Baake, Strungaru and Terauds. So when both had a similar result it is interesting to ask, if one theorem is stronger or maybe are they completely different?

# Wednesday

## The generalized nodal deficiency on metric graphs

Gilad Sofer (Israel Institute of Technology)

Sturm's oscillation theorem says that the  $n$ th eigenfunction of a Sturm-Liouville operator on an interval  $[a, b]$  has exactly  $n - 1$  zeros within  $(a, b)$ . Equivalently, it says that the zeros of the  $n$ th eigenfunction partition the interval  $[a, b]$  into exactly  $n$  connected components, called **nodal domains**.

Sturm's theorem is no longer true when moving to Laplacian eigenfunctions on metric graphs, and the amount by which a given eigenfunction violates Sturm's theorem is known as the **nodal deficiency**.

What if instead of partitioning the graph at the zeros of an eigenfunction we choose to partition it at the eigenfunction's extreme points? More generally, what if we choose to partition the graph at all points such that the eigenfunction satisfies  $\frac{f'(x)}{f(x)} = s$  for some fixed value of  $s$ ? These partitions give a generalized notion of nodal domains, which can be used to define a generalized nodal deficiency.

We show that this generalized nodal deficiency can be expressed via the Morse index of a certain linear map, which may be thought of as a generalization of the Dirichlet to Neumann map. This result generalizes an analogous index formula proven for domains in [1, 2]

The talk is based on a joint work with Ram Band and Marina Prokhorova.

### REFERENCES

- [1] G. Berkolaiko, G. Cox, J. Marzuola, *Nodal deficiency, spectral flow, and the Dirichlet-to-Neumann map*, Letters in Mathematical Physics (2019)
- [2] G. Cox, C. Jones, J. Marzuola, *Manifold decompositions and indices of Schrödinger operators*, Indiana University Mathematics Journal (2017)

## Diophantine approximation and linear repetitivity in cut and project sets

Johannes Happich (University of Leipzig)

When comparing the complexity of different aperiodic quasicrystals, it appears that linear repetitivity is a useful property that only applies to the - in some sense - most regular quasicrystals. In the talk, we will focus on a specific class of cut and project sets. We will follow the work of Haynes, Koivusalo and Walton who built a framework to reformulate questions about the distribution of patches in cut and project sets in such a way that those questions become accessible to methods of diophantine approximation. This allows a complete characterisation of linear repetitivity in terms of diophantine properties of the parameters that define the cut and project set. In the lowest possible dimensions, namely for sturmian sequences, this characterisation turns out to be rather simple. Conveniently, the same principles that work for sturmian sequences can be generalised to higher

dimensions. However, in the latter case, there are some more subtle phenomena that have to be taken into account.

**Optimal Hardy inequality for fractional Laplacians on the integers**

Marius Nietschmann (University of Potsdam)

We study the fractional Hardy inequality on the integers. We prove the optimality of the Hardy weight and hence affirmatively answer the question of sharpness of the constant. This is a joint work with Matthias Keller.

**Optimal Hardy inequalities for the discrete Heisenberg group**

Philipp Hake (University of Leipzig)

TBA

**Strictly irreducible Markov shifts and ergodic theorems for Markov groups**

Elias Zimmermann

Ergodicity criteria for skew products over Markov shifts play an important role for obtaining pointwise ergodic theorems for finitely generated groups. In this talk we extend a criterion due to Bufetov for finite state Markov shifts to Markov shifts with countable state space and show that it is in fact a characterization. The talk is based on joint work with Felix Pogorzelski.

**The boundary of a window of a cut and project scheme and asymptotic pairs for Toeplitz words**

Franziska Sieron (Friedrich-Schiller-University Jena)

TBA

# Thursday

## **$L^p$ positivity preserving**

Simon Murman (Friedrich-Schiller-University Jena)

In 2002 Braverman, Milatovic and Shubin conjectured that a complete Riemannian manifold is  $L^2$ -Positivity Preserving. That means if  $u \in L^2(M)$  and  $(-\Delta + 1)u \geq 0$  in the distributional sense then  $u \geq 0$  almost everywhere. The conjecture and its generalization to  $p \in [1, \infty]$  were proven by Güneysu, Bianchi and Setti under an additional assumption on the geometry of the manifold by proving the existence of good cut-off function. Last year Pigola and Veronelli were able to prove the conjecture for  $p \in (1, \infty)$  without the additional assumptions by a different approach. The goal of this talk is to discuss their proof.

## **A nonlinear characterization of stochastic completeness**

Ian Zimmermann (Friedrich-Schiller-University Jena)

We begin with a brief introduction to stochastic completeness, including an overview of some of the established linear characterizations of stochastic completeness of graphs. The goal of the talk is to discuss nonlinear variants of two of these. The first concerns a Liouville type property. The second involves a nonlinear resolvent.

## **Florian Fischer (University of Potsdam)**

Optimal  $p$ -Hardy weights on locally finite graphs

A natural classification of random walks is the one into recurrent and transient ones. This is equivalent to the non-/validity of the Hardy inequality for the energy functional associated with the Laplace operator on the graph. The latter is an abstract inequality between functionals and can be generalised further. In this talk, we discuss a generalisation to the quasi-linear setting and show a method to get optimal Hardy weights. We illustrate this method on the natural numbers and on regular trees. If the time permits, we also discuss the proof. The talk is based on work in progress.

## **Oleksiy Köpp (Friedrich-Schiller-University Jena)**

Martin boundary and hyperbolic graphs

TBA

## **Matti Richter (University of Potsdam)**

## An Introduction to Type Theory

Type theory refers to a type of formal system in which every term has an associated type. There are many different type theories, and some may serve as an alternative to set theory as a foundation for mathematics. Moreover, the ideas behind type theory may provide a new perspective of viewing theorems, proofs or mathematics in general. The talk aims to give an introduction to the topic for mathematicians. We will explore some core ideas of type theory, in particular the Curry-Howard correspondence. Moreover, we will take a look at function types as well as inductive and coinductive types.

### **Reflective Graphs, Ollivier curvature, effective diameter, and rigidity**

Florentin Münch (MPI, Leipzig)

We give a discrete Bonnet Myers type theorem for the effective diameter assuming positive Ollivier curvature. We prove that this diameter bound is attained if and only if the graph is a cocktail party graph, a Johnson graph, a halved cube, a Schläfli graph, a Gosset graph, or a cartesian product of the mentioned graphs with same Ollivier curvature. As a key step in the proof, we introduce the notion of reflective graphs as graphs such that for any two neighbors there exists a certain self-inverse automorphism mapping one neighbor to another. We classify these graphs as arbitrary cartesian products of the graphs mentioned before.