Program

Azzali, Sara (Potsdam)

"Trace defect formulae for classical pseudodifferential operators on covering spaces"

Abstract: A regular covering $\tilde{M}$ of a closed Riemannian manifold M is a particular example of a manifold of bounded geometry. On the (in general noncompact) covering $\tilde{M}$ it is natural to study pseudodifferential operators which are invariant by the action of the discrete group Γ of covering transformations. In this setting, Shubin, Kordyukov and many authors developed pseudodifferential calculi which are used to study the analytic and geometric properties of the space.

We discuss the properties of two local linear forms, the Wodzicki residue and the canonical trace, which are naturally defined in this setting of Γ-invariant operators. In particular, after recalling the classical Kontsevich-Vishik trace-defect formulae on a closed manifold, we will use locality properties to derive the analogous trace-defect formulae on the covering. This can be applied to give an alternative proof of Atiyah's L^2 index theorem. Based on joint work with Sylvie Paycha.

Bär, Christian (Potsdam)

"An index theorem for compact Lorentzian manifolds with boundary"

Abstract: We show that the Dirac operator on a compact globally hyperbolic Lorentzian spacetime with spacelike Cauchy boundary is a Fredholm operator if appropriate boundary conditions are imposed. We prove that the index of this operator is given by the same expression as in the index formula of Atiyah-Patodi-Singer for Riemannian manifolds with boundary. The index is also shown to equal that of a certain operator constructed from the evolution operator and a spectral projection on the boundary.

This is the first index theorem for Lorentzian manifolds and, from an analytic perspective, the methods to obtain it are quite different from the classical Riemannian case. This is joint work with Alexander Strohmaier.

Bei, Francesco (Berlin)

"On the L^2-$\overline{\partial}$-cohomology of Saper metrics"

Abstract: Saper metrics are an interesting class of complete Kähler metrics of finite volume introduced by L. Saper on the regular part of a complex projective variety V with isolated singularities. Their importance, as proved by Saper, lies in the fact that the L^2-de Rham cohomology of the regular part of V is isomorphic to the middle perversity intersection cohomology of V and that the L^2-Dolbeault cohomology is isomorphic to the Dolbeault cohomology of a resolution of V. Subsequently the construction of Saper

metrics has been generalized by Grant Melles and Milman to the case of an arbitrary subvariety Z of a compact Kähler manifold M. The goal of this talk is to describe a recent joint work with Paolo Piazza concerning the L^2-Dolbeault cohomology of Saper metrics. In particular, we have proved an analogous result of that proved by Saper in the case of isolated singularities: the L^2-Dolbeault cohomology of the regular part of Z endowed with a Saper metric as constructed by Grant Melles and Milman is isomorphic to the Dolbeault cohomology of a resolution of Z.

Fedosova, Ksenia (Bonn)

"Analytic torsion and the volume of hyperbolic manifolds "

Abstract: The talk will be about the asymptotics of the analytic torsion of hyperbolic manifolds and orbifolds. The goal is to explain how the volume can be recovered from the asymptotics, following the work of Müller, Pfaff, and myself.

Georgescu, Magdalena (Victoria)

"Spectral flow from different angles"

Abstract: In the context of bounded operators on a Hilbert space, spectral flow counts the net number of eigenvalues which change sign as one travels along a path of self-adjoint Fredholm operators. This idea can be generalized both to the context of unbounded operators and to that of a von Neumann algebra equipped with a suitable trace. Spectral flow can be used to calculate the pairing between K-theory and K-homology, making it of interest in the study of noncommutative geometry. I will discuss a few different definitions and interpretations of spectral flow, and conclude with a characterization of spectral flow for type II factors.

Gramsch, Bernhard (Mainz)

"Extensions of complex analysis for some infinite dimensional holomorphy regions and Frèchet operator algebras"

Abstract: Common work with W. Kaballo is presented. Since the space of scalar valued holomorphic functions on regions in the Schwartz distribution space $\mathcal{S}'$ is a nuclear Fréchet space we extend the homotopy principle of Grauert and Bungart to this situation.

Theorem *Let $c : X \to M$ be a continuous map from a holomorphy region X of $\mathcal{S}'$ into the set M of Fredholm operators (with fixed dimension of the kernel) in the Hörmander class (1967) of pseudo differential operators. Then c can be joined by a path in $C(X, M)$ to a holomorphic map $h : X \to M$.*

The space $\mathcal{S}'$ is of special interest in the probability theory (cf. (5)) since it has a

basis. Lempert (cf. [1]) already considered the case of holomorphy regions in Banach spaces with an unconditional basis . For finite dimensional regions the above result has been proved in [3]. The theorem is only a special consequence of a nonabelian cohomology theory on X with values in suitable operator algebras. The results and methods of [4] can be adapted to the regions $X \subset \mathcal{S}'$. The Gromov theory ([2], ch 5,6) leads to a series of interesting questions in the infinite dimensional case.

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Hänel, André (Hannover)

"Spectral asymptotics for an elastic strip with an interior crack"

Abstract: We investigate the spectral behaviour of a matrix-differential operator arising in elasticity theory. Considering an elastic strip $\Omega := \mathbb{R} \times (-\frac{\pi}{2}, \frac{\pi}{2})$ with a crack $\Gamma_\ell := [-\ell, \ell] \times \{0\}$ we search for nontrivial solutions $u : \Omega \backslash \Gamma_\ell \to \mathbb{C}^2$ of the eigenvalue problem

$$\begin{cases} (-\mu\Delta - (\lambda+\mu)\,\text{grad div})\,u &= \omega(\ell)u \qquad \text{in } \Omega\backslash\Gamma_\ell, \\ \lambda\,\text{div}(u)\cdot\vec{n} + 2\mu\varepsilon(u)\cdot\vec{n} &= 0 \qquad \text{on } \partial(\Omega\backslash\Gamma_\ell). \end{cases}$$

Here $\vec{n}$ is the outer normal unit vector, u is the displacement field of the elastic material and

$$\varepsilon(u) = \frac{1}{2}(\partial_i u_j + \partial_j u_i)_{i,j=1,2}$$

the strain tensor. Moreover, μ, λ are material coeffients, which satisfy $\mu, \lambda + \mu > 0$. We note that in general the eigenvalue problem will not admit square-integrable solutions. However, in the special case $\mu = 1$ and $\lambda = 0$ we obtain the existence of two eigenvalues $\omega_1(\ell)$ and $\omega_2(\ell)$ embedded in the essential spectrum of a corresponding self-adjoint

operator. In the present talk we prove that these eigenvalues satisfy the asymptotic estimate

$$\begin{aligned} \omega_1(\ell) &= \Lambda - \nu_1 \ell^4 + \mathcal{O}(\ell^5) \qquad \text{as} \quad \ell \to 0, \\ \omega_2(\ell) &= \Lambda - \nu_2 \ell^8 + \mathcal{O}(\ell^9) \qquad \text{as} \quad \ell \to 0, \end{aligned}$$

where Λ is some spectral threshold and $\nu_1, \nu_2 > 0$. The proof is based on a resolvent expansion of the unperturbed problem near the spectral threshold Λ and on an analysis of a corresponding Dirichlet-to-Neumann operator.

Hagger, Raffael (Hannover)

"On Limit Operators and Essential Norms"

Abstract: A classical problem in operator theory is to identify all compact operators acting on a particular Banach space. In many function spaces compactness has something to do with how the operator behaves at the boundary. For example, as is well-known, if we consider the space $L^2_a(U)$ of analytic square-integrable functions on a bounded domain $U \subset \mathbb{C}$ (with not too crazy boundary, say) and a function f contained in $C(\overline{U})$, then the corresponding Toeplitz operator T_f is compact if and only if f vanishes at the boundary. Another example is the sequence space $\ell^2(\mathbb{Z})$. An operator on $\ell^2(\mathbb{Z})$ is compact if and only if A is in BDO^2 (a certain subalgebra of $\mathcal{L}(\ell^2(\mathbb{Z}))$ to be defined within the talk) and the matrix entries $A_{i,j}$ of the corresponding infinite matrix tend to 0 towards infinity, i.e. $A_{i+k,j+k} \to 0$ as $k \to \pm\infty$ for all $i, j \in \mathbb{Z}$. Often, essential quantities like the essential spectrum or the essential norm are encoded at the boundary as well. However, the question arises what happens if the limits towards the boundary do not exist. We will address this question in the sequence spaces $\ell^p(\mathbb{Z})$ and present some recent results like

$$\|A\|_e = \max_{x \in \beta\mathbb{Z} \setminus \mathbb{Z}} \|A_x\|$$

for $A \in \mathrm{BDO}^p$, where $p \in (1, \infty)$, $\|\cdot\|_e$ denotes the essential norm of an operator, $\beta\mathbb{Z}$ denotes the Stone-Čech compactification of $\mathbb{Z}$ and the operators A_x are the limit operators of A, which will be defined within the talk. We will also discuss if and how these results can be generalized to different spaces. Most of the results presented in this talk can be found in [1].

Literatur

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Hartung, Tobias (London)

"Zeta-functions of Fourier Integral Operators: the generalized Kontsevich-Vishik trace"

Abstract: Zeta-functions are an important tool in defining traces and determinants on operator algebras. As such, they provide means of computing geometric, topological, spectral, and physical invariants. In particular, traces related to ζ-functions of pseudo-differential operators have been widely successful and are well-studied. However, Fourier Integral Operators often replace pseudo-differential operators such as it is the case in quantum field theories on curved space-time or with higher order invariants like wave-trace invariants.

In this talk, I will introduce the notion of gauged poly-log-homogeneous distributions which will enable us to compute the Laurent expansion of *zeta*-functions of gauged Fourier Integral Operators. This result will furthermore yield a generalization of the Kontsevich-Vishik trace for Fourier Integral Operators.

He, Daoyin (Göttingen)

"Critical exponents for semilinear generalized Tricomi equations"

Abstract: We consider the semilinear generalized Tricomi equation

$$\partial_t^2 u - t^m \Delta u = |u|^p$$

in $\mathbb{R}_+ \times \mathbb{R}^n$ with suitable initial data $(u(0,\cdot), \partial_t u(0,\cdot)) = (u_0, u_1)$, where $n \geq 3$ and $m \in \mathbb{N}$. The main objective of this talk is to determine the critical exponent $p_c = p_c(m,n)$ such that, for $p > p_c$, the global existence of small data solutions is guaranteed, while, for $1 < p \leq p_c$ and (non-zero) non-negative initial data, the local solution blows up in finite time.

This problem, for the wave equation when $m = 0$, was known as the *Strauss conjecture* and was resolved during the last three decades. From a physics point of view, the Tricomi equation is closely related to the study of gas flow near the sonic speed. More specifically, the Tricomi equation describes the transition from subsonic flow (in the elliptic region) to supersonic flow (in the hyperbolic region). So far, in joint work with Ingo Witt and Huicheng Yin, we have found the precise value of the critical exponent p_c. As turns out, the Tricomi operator behaves much like the wave operator. Therefore, non-weighted and weighted Strichartz estimates can established what ultimately leads to global existence. For the blowup part, one uses a suitable formula for a modified Bessel function together with the test function method.

Iwai, Toshihiro (Kyoto)

"Change in energy eigenvalues against parameters through Dirac equations with boundary conditions"

Abstract: A topological characterization of energy-band rearrangements against parameters for molecular problems with slow/fast variables comes around to a study of Dirac equations with parameters. In this talk, the Dirac equation of space-dimension three is studied under both the APS and the chiral bag boundary conditions, where the mass is viewed as a parameter ranging over all real numbers. The APS (an abbreviation of Atiyah-Patodi-Singer) boundary condition requires that eigenstates evaluated on the boundary should belong to the subspace of eigenstates associated with positive or negative eigenvalues for a boundary operator, and the chiral bag boundary condition requires that eigenstates evaluated on the boundary have chiral components related by a unitary operator. The spectral flow for a one-parameter family of operators is the net number of eigenvalues passing through zeros in the positive direction as the parameter runs. It is shown that the spectral flow for the Dirac equation with APS boundary condition is $1 + (-1) = 0$, since two eigenvalues simultaneously cross at zero energy in opposite directions, where the direction depends on the sign of spin. A counterpart of the spectral flow in the case of the chiral bag boundary condition is treated as an extension of spectral flow. In addition, discrete symmetry is discussed to explain the pattern of eigenvalues as functions of the parameter. In addition, a topological change corresponding to the spectral flow, which is observed in the corresponding semi-quantum models will be touched upon. Further, these results will be compared with the change in eigenvalues for the Dirac equation of space-dimension two under the APS and the chiral bag boundary conditions, and with a topological change observed in the corresponding semi-quantum model.

Krüger, Matthias (Göttingen)

"On the Cauchy problem for a class of degenerate hyperbolic equations"

Abstract: We study the Cauchy problem for a class of hyperbolic equations degenerate at one point of the initial hypersurface. These equations take the form

$$D_t^m u + \sum_{j=1}^{m} a_j(t,x,D_x) D^{m-j} u = f(t,x), \quad (t,x) \in (0,T) \times \mathbb{R}^d,$$

where the $a_j(t,x,D_x)$ for $1 \le j \le m$ are ΨDOs satisfying the symbol estimates

$$|\partial_t^k \partial_x^\alpha \partial_\xi^\beta a_j(t,x,\xi)| \lesssim \langle \xi \rangle^{2+2k+|\alpha|-|\beta|} \langle \sigma \langle \xi \rangle \rangle^{j-2-|\alpha|-2k}$$

with

$$\sigma(t,x) = \begin{cases} \sqrt{t+|x|^2}, & t+|x|^2 \le 1/2, \\ 1, & t+|x|^2 \ge 1. \end{cases}$$

The prototypical example is

$$\partial_t^2 u - \sigma^2(t,x)\,\Delta u = f(t,x),$$

which arises in certain models in fluid dynamics.

By a symbolic approach to obtain the energy estimates, we shall prove well-posedness for these Cauchy problems in an adapted scale of Sobolev spaces.

Kumano-go, Naoto (Kogakuin University, Japan)

"Phase space Feynman path integrals of higher order parabolic type with general functional as integrand"

Abstract: We give a general class of functionals for which the phase space Feynman path integrals of higher order parabolic type have a mathematically rigorous meaning. More precisely, for any functional belonging to our class, the time slicing approximation of the phase space path integral converges uniformly on compact subsets with respect to the endpoint of position paths and to the starting point of momentum paths. Our class of functionals is rich because it is closed under addition and multiplication. The interchange of the order with the integration with respect to time, the interchange of the order with a limit and the perturbation expansion formula hold in the path integrals.

Lescure, Jean-Marie (Aubiere)

"Fourier integral operators on Lie groupoids. (Joint work with Stèphane Vassout) "

Abstract: I will explain how to develop a calculus of Fourier integral operators on a Lie groupoid G. This begins by the study of convolability and invertibility of Lagrangian submanifolds of the symplectic groupoid T^*G, as well as their relationship with equivariant families of homogeneous canonical relations in the appropriate fibers. This allows to select a class of Lagrangian distributions on Lie groupoids that deserve the name of Fourier integral G-operators (G-FIO). By construction, the class of G-FIO contains the class of equivariant families of ordinary Fourier integral operators on the manifolds G_x, x running through the unit space. I will then explain how to adapt for G-FIO the first stages of the calculus in the spirit of Hormander's work. Finally, I will work out some examples proving the efficiency of the present approach for studying Fourier integral operators in the setting of singular manifolds.

Liu, Yingbo (Göttingen)

"Small data solutions of 2-D quasilinear wave equations with null conditions"

Abstract: For 2-D quasilinear wave equations with coefficients depending on ∂u and small initial data satisfying one or both null conditions, S. Alinhac (1999) proved a blowup and a global existence result, respectively. In this talk, we will study more general 2-D quasilinear wave equations with coefficients depending on u and ∂u. Through the construction of an approximate solution, combined with weighted energy estimates, a blowup and a global existence result are established by continuous induction.

Namboodiri, M.N.N. (Kochi)

"Čhebešev subspaces and Toeplitz C^*-algebras"

Abstract: The study of Čebyšev subspaces in the general operator algebra setting was initiated by A. G. Robertson [4] followed by Robertson and Yost [5] and then Pedersen [3]. In [4], Robertson gives a characterization of one dimensional Čebyšev subspaces of von Neumann algebras. Generalizations of this result to arbitrary finite dimensions can be found in [1] where a non-commutative version of the classical Haar criterion is introduced. This lecture reports the work in progress jointly with W. Bauer about a study of these developments in the context of Toeplitz C^*-algebras, especially Haar criterion, classical as well as non-commutative.

Acknowledgement: I am thankful to UGC India for providing Emeritus Fellowship at CUSAT. I am also thankful to the organizers of this wonderful workshop for kindly extending an invitation in 2016 again.

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Ruan, Zhuoping (Göttingen)

"On the existence of solutions of semilinear Tricomi-type equations"

Abstract: We investigate solutions $u = u(t, x)$ of semilinear Tricomi-type equations

$$u_{tt} - t^m \Delta u = F(t, x, u), \quad (t, x) \in \mathbb{R} \times \mathbb{R}^n, \tag{1}$$

where $m \in \mathbb{N}$. The talk consists of three parts:

1. In the degenerate hyperbolic domain $\mathbb{R}_+ \times \mathbb{R}^n$, we first consider the local existence and singularity structure of low regularity solutions u of Eq. (1) with a $\mathscr{C}^\infty$ nonlinear term F, $u(0, \cdot) = 0$, and typical discontinuous initial data $u_t(0, \cdot)$ (homogeneous of degree zero or piecewise smooth).

2. For m odd, in the mixed elliptic-hyperbolic domain $\mathbb{R} \times \mathbb{R}^n$, we are then concerned with low regularity solutions u of Eq. (1) with a $\mathscr{C}^1$ nonlinear term F and initial data $u(0, \cdot) \in H^s(\mathbb{R}^n)$.

3. Again in $\mathbb{R}_+ \times \mathbb{R}^n$, we finally study the sharp local existence of minimal regularity solutions u of Eq. (1) and the global existence of small H^s data solutions u in the superconformal case.

This is joint work with Ingo Witt and Huicheng Yin.

Savin, Anton and **Sternin, Boris** (Peoples Friendship University of Russia)

"Elliptic problems associated with diffeomorphisms of manifolds with boundary"

Abstract: The aim of this work is to study a new class of boundary value problems on compact manifolds with boundary, in which the main and the boundary operators are nonlocal and associated with smooth mappings of the manifold to itself. This setting includes a number of well-known problems as special cases. Namely, problems for invertible mappings (i.e., diffeomorphisms), see [1]; problems with dilations in $\mathbb{R}^n$, see [2]. Finally, we mention well-known Bitsadze–Samarskii problems [3], in which boundary values of functions are related with their values on a submanifold lying inside the domain.

As a main result of this talk we obtain conditions, which guarantee the Fredholm property of the class of operators in question provided that the operators are associated with *compressions* of the manifold (compressions are mappings of manifolds with boundary, which map the manifold strictly inside itself). Finding an analogue of Shapiro–Lopatinskii conditions in our setting is the most important part of the work.

It turns out that in the case of nonlocal problems associated with compressions, the ellipticity condition has a fundamentally new form. The point here is that for a problem

with compressions it is necessary to freeze the coefficients of the operator not only at boundary points but on their entire orbit under the compression and its iterations. As a result, the condition of Shapiro–Lopatinskii type requires unique solvability of an *infinite* matrix system of ordinary differential equations, which corresponds to the trajectories of points on the boundary.

The work was partially supported by RFBR grants Nos 15-01-08392 and 16-01-00373.

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Schrohe, Elmar (Hannover)

"Spectral flow of exterior Landau-Robin hamiltonians"

Abstract: We study the spectral flow of Landau-Robin hamiltonians in the exterior of a compact domain with smooth boundary. This provides a method to study the spectrum of the exterior Landau-Robin hamiltonian's dependence on the choice of Robin data, even explaining the heuristics of how the spectrum of the Robin problem asymptotically tends to the spectrum of the Dirichlet problem. The main technical result concerns the continuous dependence of Landau-Robin hamiltonians on the Robin data in the gap topology. The problem can be localized to the compact boundary where the asymptotic behavior of the spectral flow in some special cases can be described.

Joint work with Magnus Goffeng. Preprint arXiv:1505.06080. To appear in J. Spectral Theory

Schulze, Bert-Wolfgang (Potsdam)

"Volterra Mellin Operators"

Abstract: The results of this presentation belong to a joint project [7] with M. Hedayat

Mahmoudi (Potsdam) and R. Schulz (Hannover) on parabolic operators on manifolds with edge, see also the joint article on anisotropic edge calculus with R. Schulz [6].

It has been known for a long time, cf. [9], that ellipticity of a pseudo-differential operator on a manifold M with edge Y is determined by a principal symbolic hierarchy and a notion of ellipticity. Similarly as [1] there is a calculus of 2×2 block matrices containing parametrices of elliptic elements, and for compact M ellipticity and Fredholm property in weighted edge spaces are equivalent. The typical differential operators on M are edge-degenerate and studied together with edge conditions. Those play a similar role as boundary conditions in the special case of a manifold with boundary. Such conditions are expressed in terms of trace and potential operators, and there are also Green operators in the upper left corners of block matrices. Parametrices produce elliptic regularity of solutions with asymptotics at the edge, especially through the ideal of smoothing Mellin plus Green operators. In this connection we have specific si ngular f unctions of edge asymptotics belonging to weighted edge Sobolev spaces.

A similar program in the context of parabolicity requires a new inspection of the above-mentioned "isotropic" edge calculus, now to make it work for symbols containing the time covariable τ and spatial covariables ξ of different order. Most classical in the non-singular situation is the heat operator $\partial_t - \Delta$, with t being the time variable and Δ the Laplacian, say, on a Riemannian manifold M. One of the main issues is to coordinate edge quantizations with Volterra property and parabolicity. The right way appeared mysterious for a long time, since the Volterra property does not survive under common edge quantizations. The general motivation of the present investigation is an anisotropic pseudo-differential calculus on manifolds with edge containing parabolic differential operators together with their inverses in the class of Volterra edge operators. On a time-spatial cylinder where the cross section is a smooth compact manifold su ch a cal culus has been elaborated by Piriou in [8]. This structure is recovered also for the calculus over $M \setminus Y$ when M is a manifold with edge Y. Close to the edge we have an anisotropic principal symbolic hierarchy, in particular, the principal twisted homogeneous edge symbol, acting in Kegel spaces, and we introduce parabolicity and Volterra property of such operator families. In particular, we illustrate the specific novelties in the case of the heat operator on a wedge, and we develop further new elements of the Volterra edge calculus.

Literatur

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Seiler, Jörg (Torino)

"Elliptic complexes with generalized Atiyah-Patodi-Singer boundary conditions"

Abstract: Let M be a smooth, compact manifold with boundary. Consider a complex

$$\mathfrak{A}: \ 0 \longrightarrow H^s(M, E_0) \xrightarrow{A_0} H^{s-\nu_0}(M, E_1) \xrightarrow{A_1} \ldots \xrightarrow{A_n} H^{s-\nu_n}(M, E_{n+1}) \longrightarrow 0$$

of (pseudo)differential operators A_j acting in Sobolev spaces of sections in vector-bundles, which is exact on the level of homogeneous principal symbols. Here, $\nu_j = \mu_0 + \ldots + \mu_j$ where μ_j is the order of A_j.

In the short note [1] Dynin announced the following result:

Theorem. The following are equivalent:

a) There exist trace operators $T_j : H_j \to L_j$ and pseudodifferential operators $Q_j : L_j \to L_{j+1}$ on the boundary, where

$$H_j := H^{s-\nu_{j-1}}(M, E_j), \qquad L_j := H^{s-\nu_j}(\partial M, F_{j+1}),$$

such that the boundary value problem

$$\begin{array}{ccccccccccc} 0 & \longrightarrow & H_0 & \xrightarrow{A_0} & H_1 & \xrightarrow{A_1} & \dots & \xrightarrow{A_n} & H_{n+1} & \longrightarrow & 0 \\ & & \downarrow{\scriptstyle T_0} & & \downarrow{\scriptstyle T_1} & & & & \downarrow{\scriptstyle T_{n+1}} & & \\ 0 & \longrightarrow & L_0 & \xrightarrow{Q_0} & L_1 & \xrightarrow{Q_1} & \dots & \xrightarrow{Q_n} & L_{n+1} & \longrightarrow & 0 \end{array}$$

is Fredholm (i.e., the associated mapping cone has finite-dimensional cohomology spaces) and such that every block-matrix operator $\begin{pmatrix} -A_j & 0 \\ T_j & Q_{j-1} \end{pmatrix}$ belongs to Boutet de Monvel's calculus.

b) The complex $\mathfrak{A}$ satisfies the so-called Atiyah-Bott condition.

Apparently a proof of this result has never been published. Not only we give a rigorous proof, but we also provide a substantial extension of this theorem: To any complex $\mathfrak{A}$ as above (possibly violating the Atiyah-Bott obstruction) there exist zero-order pseudodifferential projections P_j acting on sections in F_j such that a replacement of T_j by P_jT_j, of Q_j by $P_{j+2}Q_jP_{j+1}$, and of L_j by $\operatorname{im} P_{j+1} = P_{j+1}(L_j)$ leads to a Fredholm problem. Such boundary conditions can be viewed as an analogue of the famous Atiyah-Patodi-Singer (spectral) boundary conditions in the framework of elliptic complexes.

An essential ingredient of the proof is the concept of pseudodifferential operators of Toeplitz type [5]. This is joint work with B.-W. Schulze [3].

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Swoboda, Jan (München)

"Higgs bundle moduli spaces: geometric and analytic aspects"

Abstract: TBA

Tarama, Daisuke (Kyoto)

"Stability analysis for generalized free rigid body dynamics on real semi-simple Lie algebras"

(Joint work with Tudor S. Ratiu (EPFL))

Abstract: The free rigid body dynamics, which describes the rotational motion of a rigid body under no external force, is one of the typical solvable systems in analytical mechanics. Mathematically, this dynamics is formulated as a Hamiltonian system on the cotangent bundle $T^*SO(3)$ of the rotation group $SO(3)$. In other words, the system is the geodesic flow on $SO(3)$ with respect to a left-invariant Riemannian metric. Because of its symmetry, the motion is essentially described by the Euler equation on the Lie algebra $\mathfrak{so}(3)$. Its complete integrability and the stability property of the equilibria are well know.

Around 1980, Mishchenko and Fomenko have generalized this system to geodesic flows on arbitrary semi-simple Lie groups. Although the complete integrability of such a generalized free rigid body dynamics is investigated well, the stability of the equilibria has not been analyzed until recently.

This talk deals with the stability analysis for generic equilibria of the generalized free rigid body dynamics on arbitrary real semi-simple Lie algebras. It is, e.g., shown that the stability property, in general, depends on the choice of the Cartan subalgebra with respect to which the Riemannian metric (generalized inertia tensor) is defined.

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Tepoyan, Liparit (Yerevan)

"Degenerate nonselfadjoint high-order differential-operator equations on an infinite interval"

Abstract: We consider degenerate nonselfadjoint high-order differential-operator equation

$$Lu \equiv (-1)^m (t^\alpha u^{(m)})^{(m)} + (-1)^{m-1} A (t^{\alpha-1} u^{(m)})^{(m-1)} + t^\beta P u = f(t), \qquad (1)$$

where $t \in (1; +\infty)$, $m \in \mathbb{N}$, $\alpha \neq 1, 3, \ldots, 2m-1$, $\beta \leq \alpha - 2m$, linear operators A and P, in general unbounded, acts in a separable Hilbert space H, $f \in L_{2,-\beta}((1, +\infty), H)$. It is assumed that the operators A and P have a complete system of eigenfunctions $\{\varphi_k\}_{k=1}^{\infty}$, which form a Riesz basis in H. Under some conditions on the operators A and P we prove unique solvability of the equation (1) for every $f \in L_{2,-\beta}((1, +\infty), H)$, as well as give description of $D(L)$ for the one-dimensional equation when $A = 0, \beta = -2m$.

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Vertman, Boris (Münster)

"Ricci Flow on singular edge manifolds"

Abstract: We discuss recent existence results for a Ricci flow starting at an incomplete manifold with edge singularities and bounded Ricci curvature, flowing for a short time within a class of incomplete edge manifolds. We explain regularity properties for the corresponding family of Riemannian metrics and point out boundedness of the Riemannian curvature tensor along the flow. For Riemannian metrics that are sufficiently close to a Ricci flat incomplete edge metric, such a Ricci flow exists for all times. Our results in particular include the case of Kaehler metrics with isolated conical singularities. The proof works by a careful analysis of the Lichnerowicz Laplacian and the Ricci de Turck flow equation.