
C*-Algebras

Winter semester 2016/17

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Sheet 8

- (1) Let A be the Banach algebra $L^1([0, 1])$ with convolution as multiplication. Show that A has no non-trivial ideals.
(Hint: Show first that A can be generated by the constant functions. Secondly show that the spectral radius of every element is zero.)

- (2) Show that in a C^* -algebra we have

$$\|x\| = \sup_{\|y\| \leq 1} \|xy\|.$$

Give an example of a Banach algebra where this equality fails.

- (3) Let I be an index set and let $A_i, i \in I$, be C^* -algebras. Show

$$\bigoplus_{i \in I} A_i = \{a \in \prod_{i \in I} A_i \mid \sup_{i \in I} \|a_i\| < \infty\}$$

together with pointwise operations is a C^* -algebra.

- (4) Let A be a non-unital C^* -algebra that is generated by an element $a \in A$. Then there is a unique isometric isomorphism of involutive algebras

$$\phi : C_0(\sigma(a) \setminus \{0\}) \rightarrow A$$

such that

$$\phi(\text{id}) = a.$$